Lecture II: The evolution of dispersal

January 2017

Terminology

Dispersal "Any movement of individuals or propagules with potential consequences for gene flow across space" [Ronce, 2007]

(c) Wikimedia

Terminology

Dispersal "Any movement of individuals or propagules with potential consequences for gene flow across space" [Ronce, 2007]

(c) Wikimedia

Migration "Mass directional movements of large numbers of a species from one location to another." [Begon et al., 1996]

But in population genetics, often used as a synonym of dispersal.

(c) Wikimedia

Why disperse?

Why disperse?

- Avoid kin competition

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- Avoid kin competition
- Avoid inbreeding

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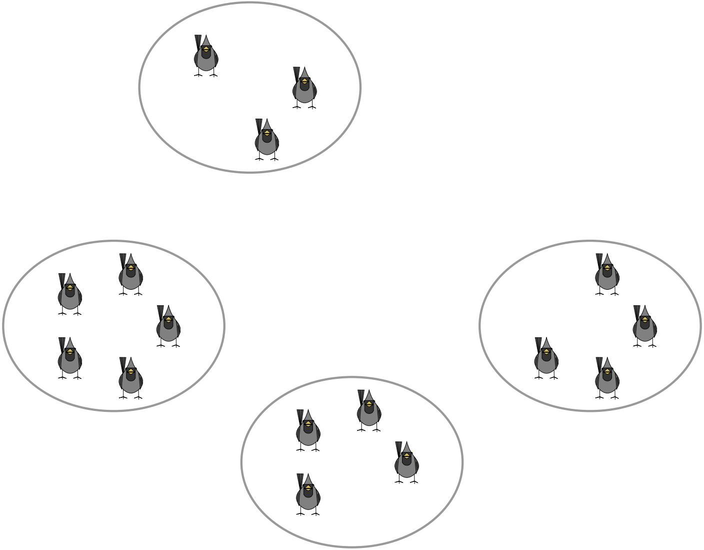
- Avoid inbreeding
- Explore new territories

Why disperse?

- Avoid kin competition
- Avoid inbreeding
- Explore new territories
- Find better conditions.

Dispersal stages



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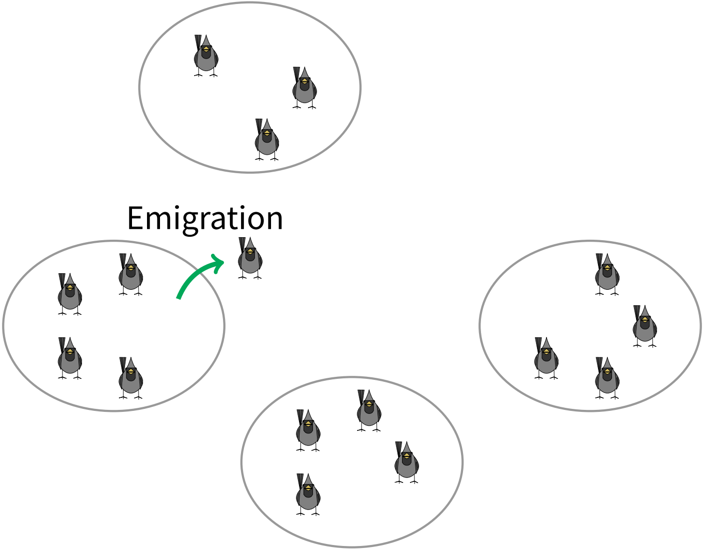

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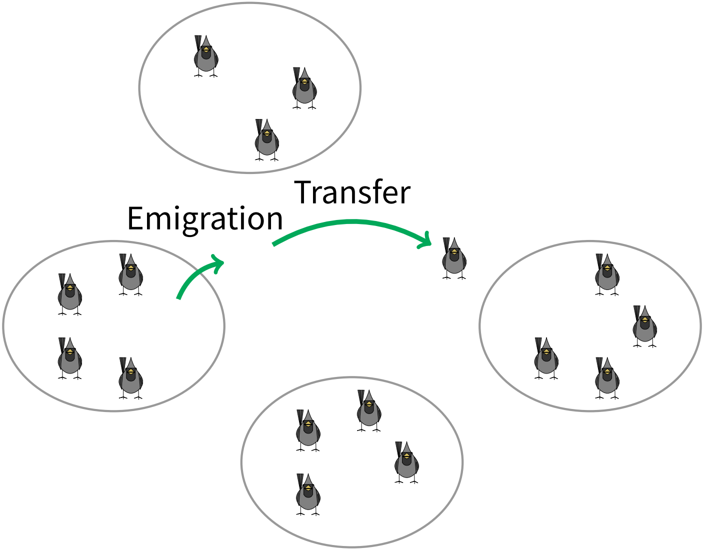

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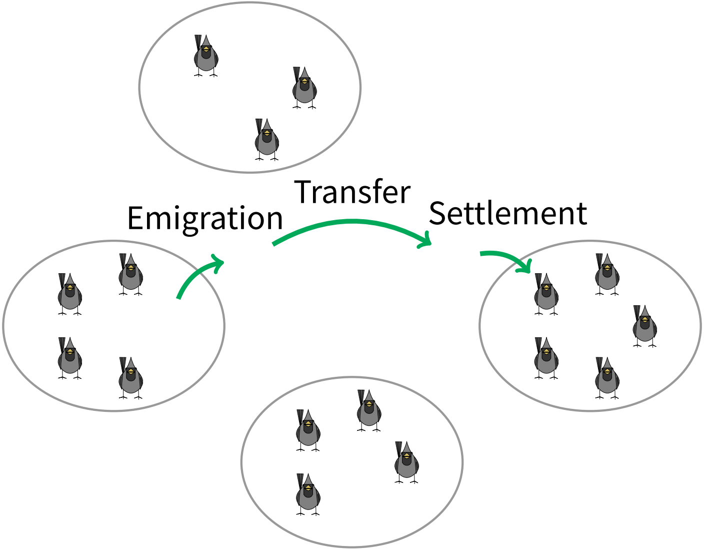

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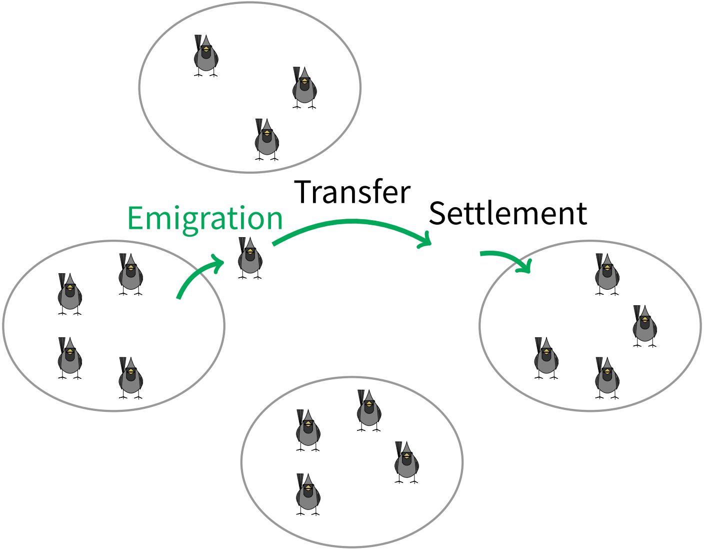


Outline

Introduction

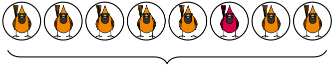
Dispersal and kin competition

- Hamilton & May 1977
- Island model

In spatially heterogeneous environments

An iconic example: [Hamilton and May, 1977]

Model



N saturated sites,
$N \to \infty$

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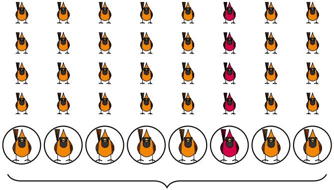

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Offspring production

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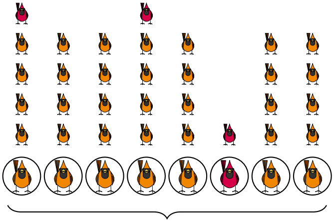

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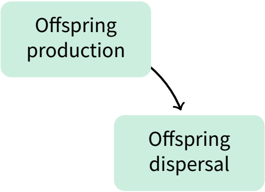

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Cost of emigration $c = 1 - p$.

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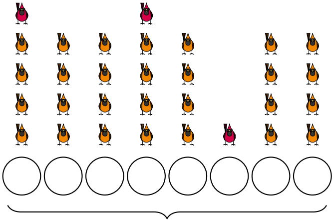

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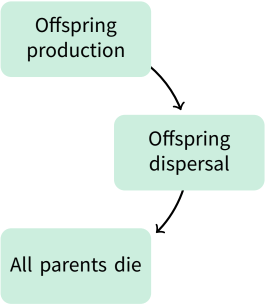

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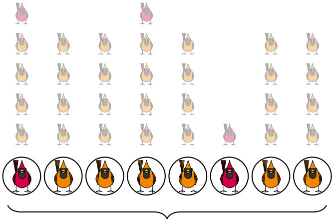

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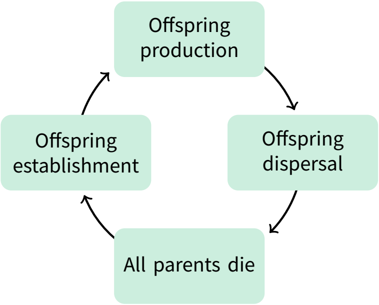

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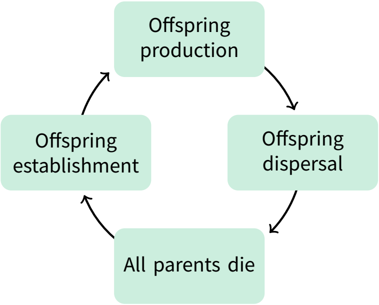

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Invasion fitness

$$w(y,x) = \frac{1-y}{1-y+(1-c)\,x} + \frac{(1-c)\,y}{1-x+(1-c)\,x}$$

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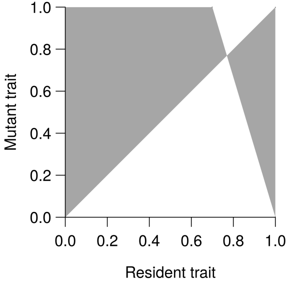
$$D(x) = \left.\frac{\partial w(y,x)}{\partial y}\right|_{y=x} = \frac{(1-c)(1-x(1+c))}{(1-cx)^2}$$

PIP



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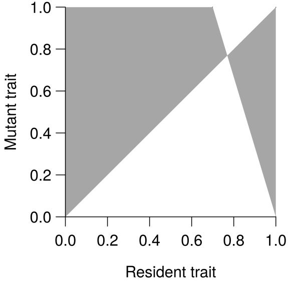

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$$\frac{dD(x)}{dx} = -\frac{(1-c)(1-c+(c+1)cx)}{(1-cx)^3} \leq 0$$

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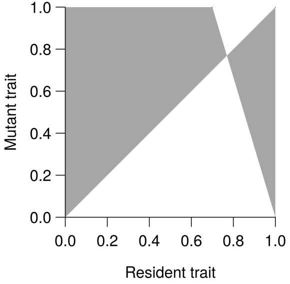

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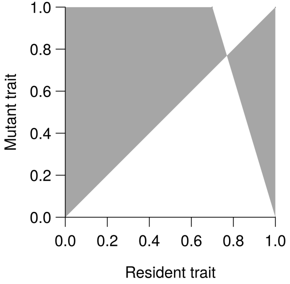
- Uninvadability

$$\left.\frac{\partial^2 w(y,x)}{\partial y^2}\right|_{y=x=x^*} = -2(1-c)(c+1)^2 \leq 0$$

PIP



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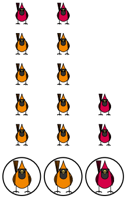
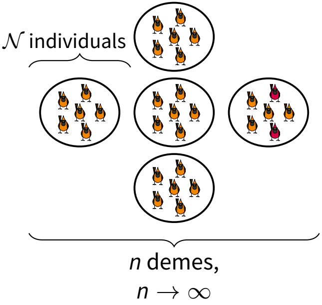
Kin competition Competition between related individuals.



Dispersal evolution in a subdivided population



z_r	Emigration probability of residents
z_m	Emigration probability of mutants
c	Cost of dispersal
μ	Mutation probability ($\mu \to 0$).

[Gandon and Rousset, 1999]

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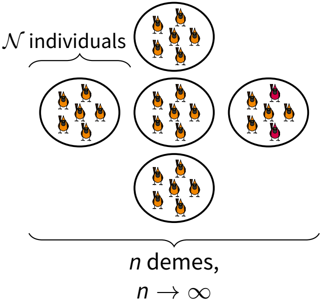

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$q_0(z_m, z_r)$: Average frequency of mutants in demes that contain mutants.

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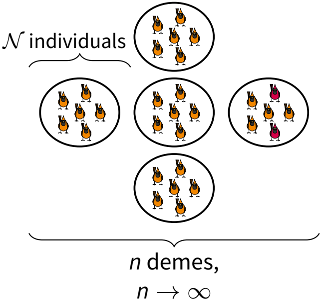

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Invasion fitness

$$w(z_m, z_r) = \frac{1 - z_m}{1 - (q_0\, z_m + (1 - q_0) z_r) + (1 - c)\, z_r} + \frac{(1 - c)\, z_m}{1 - z_r + (1 - c) z_r}$$

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Selection gradient

$$D(z) = \left.\frac{\partial w(z_m, z_r)}{\partial z_m}\right|_{z_m = z_r = z} = \frac{q - c - z\left(q - c^2\right)}{(1 - c\, z)^2},$$

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Computing q, recursively ▸ More on q

$$q_{t+1} = \frac{1}{\mathcal{N}} + \frac{\mathcal{N} - 1}{\mathcal{N}} \left(1 - \frac{(1 - c) z}{1 - cz}\right)^2 q_t$$

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Dispersal evolution in a subdivided population (3)

Singular strategy

$$z^* = \frac{1 + 2c\,\mathcal{N} - \sqrt{1 + 4\,c^2\,(\mathcal{N} - 1)\,\mathcal{N}}}{2\,c\,(1+c)\,\mathcal{N}}.$$

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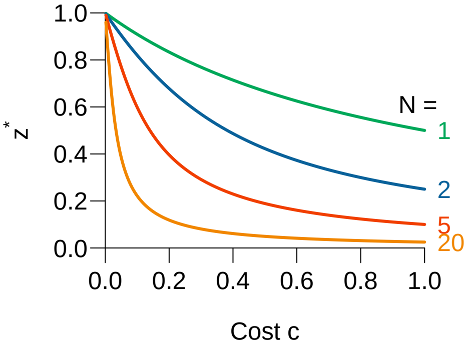

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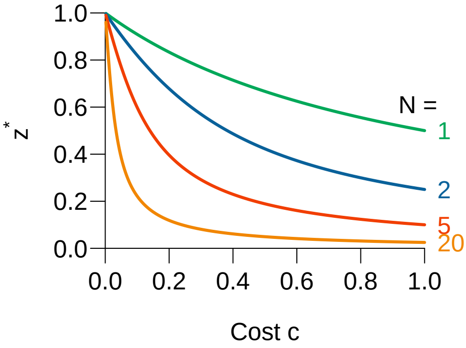

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Dispersal evolution in a subdivided population (4)

Invadability

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$$\left.\frac{\partial^2 w(z_m, z_r)}{\partial z_m^2}\right|_{z_m=z_r=z^*} =$$

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- In this model, always z^* is always uninvadable [Ajar, 2003].

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- But with heterogeneity in deme sizes, diversification can occur [Massol et al., 2011]

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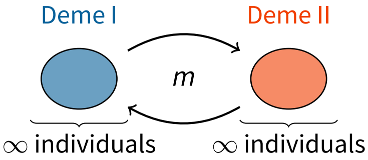
Dispersal and kin competition

In spatially heterogeneous environments

Another classic: [Balkau and Feldman, 1973]



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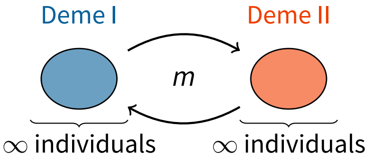

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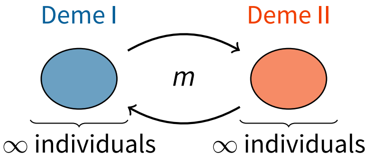

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  Fitness:

	in I	in II
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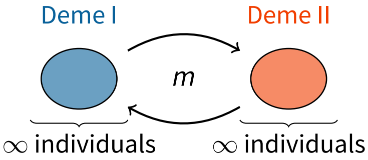

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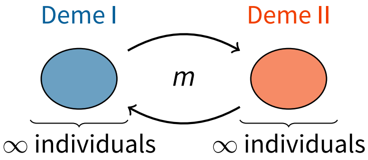

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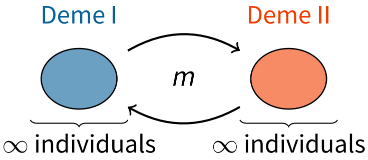

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With AB and aB

Frequency of AB is x in deme I and y in deme II.

$$x' = (1-z)\frac{(1+s)\,x}{(1+s)\,x+1-x} + z\frac{y}{y+(1+s)(1-y)}$$
$$y' = z\frac{(1+s)\,x}{(1+s)\,x+1-x} + (1-z)\frac{y}{y+(1+s)(1-y)}.$$

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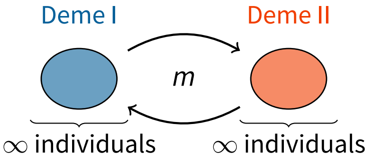

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$$y' = z\frac{(1+s)\,x}{(1+s)\,x+1-x} + (1-z)\frac{y}{y+(1+s)(1-y)}.$$

$\rightarrow$ Equilibrium $(\hat{x},\hat{y}) = (\hat{x}, 1-\hat{x})$.

Another classic: [Balkau and Feldman, 1973] (2)

Dynamics with the four genotypes

		AB	Ab	aB	ab
Frequencies:	in deme I	x_1	x_2	x_3	x_4
	in deme II	y_1	y_2	y_3	y_4

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$$x_1' = (1-z)\frac{(1+s)x_1}{(1+s)(x_1+x_2)+(x_3+x_4)} + z\frac{y1}{(y_1+y_2)+(1+s)(y_3+y_4)}$$

$$x_2' = (1-z_m)\frac{(1+s)x_2}{(1+s)(x_1+x_2)+(x_3+x_4)} + z_m\frac{y2}{(y_1+y_2)+(1+s)(y_3+y_4)}$$

$$x_3' = \ldots$$

Another classic: [Balkau and Feldman, 1973] (2)

Dynamics with the four genotypes

		AB	Ab	aB	ab
Frequencies:	in deme I	x_1	x_2	x_3	x_4
	in deme II	y_1	y_2	y_3	y_4

$$x'_1 = (1-z)\frac{(1+s)x_1}{(1+s)(x_1+x_2)+(x_3+x_4)} + z\frac{y1}{(y_1+y_2)+(1+s)(y_3+y_4)}$$
$$x'_2 = (1-z_m)\frac{(1+s)x_2}{(1+s)(x_1+x_2)+(x_3+x_4)} + z_m\frac{y2}{(y_1+y_2)+(1+s)(y_3+y_4)}$$
$$x'_3 = \dots$$

Invasion analysis

Local stability of the equilibrium without b,
$(\hat{x}, 0, 1-\hat{x}, 0, \hat{y}, 0, 1-\hat{y}, 0)$

▸ More on stability analysis

Another classic: [Balkau and Feldman, 1973] (3)

```
ev = Eigenvalues[Jac] // FullSimplify
```

$$\Bigg\{\frac{4\,(1+s)}{\left(2+s-2z-sz+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)^2},\ -\frac{4\,(1+s)\,(-1+2z)}{\left(2+s-2z-sz+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)^2},\ -\frac{4\,(1+s)\,(-1+2z)}{\left(2+s-2z-sz+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)^2},$$

$$1,\ \Bigg(4+2\sqrt{s^2-2s^2z+(2+s)^2z^2}+s^2(-1+z)(-1+zm)+4z(-1+zm)-s\left(4-4z+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)(-1+zm)-$$

$$2\left(2+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)zm-\sqrt{2}\,\sqrt{\Bigg(\Bigg(s^2(-1+z)^2+2\left(1+2z^2+\sqrt{s^2-2s^2z+(2+s)^2z^2}-z\left(2+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)\right)+}$$

$$\overline{s\left(2+4z^2+\sqrt{s^2-2s^2z+(2+s)^2z^2}-z\left(4+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)\right)\Bigg)\left(s^2-2s^2zm+(2+s)^2zm^2\right)\Bigg)}\Bigg)\Bigg/$$

$$\left(2+s-2z-sz+\sqrt{4sz+(s(-1+z)+2z)^2}\right)^2,\ \Bigg(4+2\sqrt{s^2-2s^2z+(2+s)^2z^2}+s^2(-1+z)(-1+zm)+4z(-1+zm)-$$

$$s\left(4-4z+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)(-1+zm)-2\left(2+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)zm+$$

$$\sqrt{2}\,\sqrt{\Bigg(\Bigg(s^2(-1+z)^2+2\left(1+2z^2+\sqrt{s^2-2s^2z+(2+s)^2z^2}-z\left(2+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)\right)+s\Bigg(2+4z^2+\sqrt{s^2-2s^2z+(2+s)^2z^2}-}$$

$$\overline{z\left(4+\sqrt{s^2-2s^2z+(2+s)^2z^2}\right)\Bigg)\Bigg)\left(s^2-2s^2zm+(2+s)^2zm^2\right)\Bigg)}\Bigg)\Bigg/\left(2+s-2z-sz+\sqrt{4sz+(s(-1+z)+2z)^2}\right)^2\Bigg\}$$

Another classic: [Balkau and Feldman, 1973] (3)

```
ev = Eigenvalues[Jac] // FullSimplify
```

$$
\begin{aligned}
\Bigg\{ & \frac{4\,(1+s)}{\left(2+s-2\,z-s\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)^2},\ -\frac{4\,(1+s)\,(-1+2\,z)}{\left(2+s-2\,z-s\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)^2},\ -\frac{4\,(1+s)\,(-1+2\,z)}{\left(2+s-2\,z-s\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)^2},\\
& 1,\ \Bigg(4+2\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}+s^2\,(-1+z)\,(-1+zm)+4\,z\,(-1+zm)-s\left(4-4\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)(-1+zm)-\\
& \quad 2\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)zm-\sqrt{2}\,\sqrt{\Bigg(\Bigg(s^2\,(-1+z)^2+2\left(1+2\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-z\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\right)+}\\
& \quad\quad \overline{s\left(2+4\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-z\left(4+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\right)\Bigg)\left(s^2-2\,s^2\,zm+(2+s)^2\,zm^2\right)\Bigg)}\Bigg)\Big/\\
& \quad \left(2+s-2\,z-s\,z+\sqrt{4\,s\,z+(s\,(-1+z)+2\,z)^2}\right)^2,\ \Bigg(4+2\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}+s^2\,(-1+z)\,(-1+zm)+4\,z\,(-1+zm)-\\
& \quad s\left(4-4\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)(-1+zm)-2\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)zm+\\
& \quad \sqrt{2}\,\sqrt{\Bigg(\Bigg(s^2\,(-1+z)^2+2\left(1+2\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-z\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\right)+s\Bigg(2+4\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-}\\
& \quad\quad \overline{z\left(4+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\Bigg)\Bigg)\left(s^2-2\,s^2\,zm+(2+s)^2\,zm^2\right)\Bigg)}\Bigg)\Big/\left(2+s-2\,z-s\,z+\sqrt{4\,s\,z+(s\,(-1+z)+2\,z)^2}\right)^2\Bigg\}
\end{aligned}
$$

$\rightarrow$ All eigenvalues ρ_i such that $|\rho_i| \leq 1$ when $z_m > z$

Another classic: [Balkau and Feldman, 1973] (3)

```
ev = Eigenvalues[Jac] // FullSimplify
```

$$
\begin{aligned}
\Bigg\{ & \frac{4\,(1+s)}{\left(2+s-2\,z-s\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)^2},\ -\frac{4\,(1+s)\,(-1+2\,z)}{\left(2+s-2\,z-s\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)^2},\ -\frac{4\,(1+s)\,(-1+2\,z)}{\left(2+s-2\,z-s\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)^2},\\
& 1,\ \Bigg(4+2\,\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}+s^2\,(-1+z)\,(-1+zm)+4\,z\,(-1+zm)-s\left(4-4\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)(-1+zm)-\\
&\quad 2\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)zm-\sqrt{2}\,\sqrt{\Bigg(\Bigg(s^2\,(-1+z)^2+2\left(1+2\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-z\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\right)+}\\
&\quad\quad s\left(2+4\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-z\left(4+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\right)\Bigg)\left(s^2-2\,s^2\,zm+(2+s)^2\,zm^2\right)\Bigg)}\Bigg)\Big/\\
&\quad \left(2+s-2\,z-s\,z+\sqrt{4\,s\,z+(s\,(-1+z)+2\,z)^2}\right)^2,\ \Bigg(4+2\,\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}+s^2\,(-1+z)\,(-1+zm)+4\,z\,(-1+zm)-\\
&\quad s\left(4-4\,z+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)(-1+zm)-2\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)zm+\\
&\quad \sqrt{2}\,\sqrt{\Bigg(\Bigg(s^2\,(-1+z)^2+2\left(1+2\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-z\left(2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\right)+s\left(2+4\,z^2+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}-\right.}\\
&\quad\quad \left.z\left(4+\sqrt{s^2-2\,s^2\,z+(2+s)^2\,z^2}\right)\right)\Bigg)\left(s^2-2\,s^2\,zm+(2+s)^2\,zm^2\right)\Bigg)\Bigg)\Big/\left(2+s-2\,z-s\,z+\sqrt{4\,s\,z+(s\,(-1+z)+2\,z)^2}\right)^2\Bigg\}
\end{aligned}
$$

$\rightarrow$ All eigenvalues ρ_i such that $|\rho_i| \leq 1$ when $z_m > z$

Reduced emigration probabilities are favored.

A few take-home messages

- Kin competition favors the evolution of emigration

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- Kin competition favors the evolution of emigration
- Spatial heterogeneity only does not…

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- Spatial heterogeneity only does not…
  but dispersal can evolve when local conditions change with time and space.

A few take-home messages

- Kin competition favors the evolution of emigration
- Spatial heterogeneity only does not...
  but dispersal can evolve when local conditions change with time and space.
- Dispersal is a complicated trait to study, because it affects spatial structure ($\rightarrow$ Lecture 4).

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Appendix

Outline

More on q

Stability analysis

More on q

New parameters:

n	Number of demes
μ	Mutation probability (infinite allele model)

[Cockerham and Weir, 1987]

Back

More on q

New parameters:

- n Number of demes
- μ Mutation probability (infinite allele model)
- $m = \dfrac{1-z}{1-c\,z}$ Backward dispersal probability

[Cockerham and Weir, 1987]

Back

More on q

New parameters:

n	Number of demes
μ	Mutation probability (infinite allele model)
$m = \dfrac{1-z}{1-c\,z}$	Backward dispersal probability

Probability that two individuals came from the same deme and

- are in the same deme: $a = (1-m)^2 + \frac{m^2}{n-1}$,

[Cockerham and Weir, 1987]

Back

More on q

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μ Mutation probability (infinite allele model)

$m = \dfrac{1-z}{1-c\,z}$ Backward dispersal probability

Probability that two individuals came from the same deme and

- are in the same deme: $a = (1-m)^2 + \frac{m^2}{n-1}$,
- are in different demes: $b = \frac{1-(1-m)^2}{n-1} - \frac{m^2}{(n-1)^2}$.

[Cockerham and Weir, 1987]

Back

More on q

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Probabilities of identity by descent, with replacement:

- In the same deme: $q_{0,t+1} = \frac{1}{\mathcal{N}} + \frac{\mathcal{N}-1}{\mathcal{N}}(1-\mu)^2 \left(a\,q_{0,t} + (1-a)\,q_{1,t}\right)$,

[Cockerham and Weir, 1987]

Back

More on q

New parameters:

n Number of demes

μ Mutation probability (infinite allele model)

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- In different demes: $q_{1,t+1} = (1-\mu)^2 \left(b\,q_{0,t} + (1-b)\,q_{1,t}\right)$,

[Cockerham and Weir, 1987]

Back

More on q (2)

$$
\begin{aligned}
q_{0,t+1} &= \frac{1}{\mathcal{N}} + \frac{\mathcal{N}-1}{\mathcal{N}}(1-\mu)^2\left(a\, q_{0,t} + (1-a)\, q_{1,t}\right),\\
q_{1,t+1} &= (1-\mu)^2\left(b\, q_{0,t} + (1-b)\, q_{1,t}\right),
\end{aligned}
$$

Order of limits

[Cockerham and Weir, 1987]

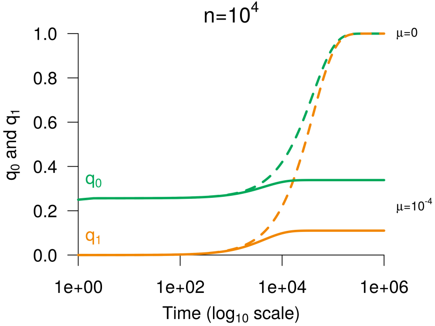
Back

More on q (2)

$$q_{0,t+1} = \frac{1}{\mathcal{N}} + \frac{\mathcal{N}-1}{\mathcal{N}}(1-\mu)^2 \left(a\, q_{0,t} + (1-a)\, q_{1,t}\right),$$
$$q_{1,t+1} = (1-\mu)^2 \left(b\, q_{0,t} + (1-b)\, q_{1,t}\right),$$

Order of limits

- When $\mu = 0$, $q_{0,\infty} = q_{1,\infty} = 1$.



[Cockerham and Weir, 1987]

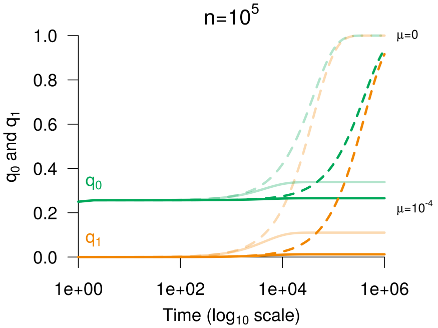
Back

More on q (2)

$$q_{0,t+1} = \frac{1}{\mathcal{N}} + \frac{\mathcal{N}-1}{\mathcal{N}}(1-\mu)^2 \left(a\, q_{0,t} + (1-a)\, q_{1,t}\right),$$
$$q_{1,t+1} = (1-\mu)^2 \left(b\, q_{0,t} + (1-b)\, q_{1,t}\right),$$

Order of limits

- When $\mu = 0$, $q_{0,\infty} = q_{1,\infty} = 1$.
- When $n \to \infty$, $q_{1,\infty} = 0$ and $q_{0,\infty} = \frac{1}{\mathcal{N}} + \frac{\mathcal{N}-1}{\mathcal{N}}(1-\mu)^2 \left(a\, q_{0,\infty}\right)$.



[Cockerham and Weir, 1987]

Back

Outline

More on q

Stability analysis

Stability analysis for discrete-time models

Model

$$
\begin{aligned}
N_1(t+1) &= G_1(N_1(t), N_2(t), \ldots, N_k(t)) \\
N_2(t+1) &= G_2(N_1(t), N_2(t), \ldots, N_k(t)) \\
&\vdots \\
N_k(t+1) &= G_k(N_1(t), N_2(t), \ldots, N_k(t))
\end{aligned}
$$

Stability analysis for discrete-time models

Model

$$
\begin{aligned}
N_1(t+1) &= G_1(N_1(t), N_2(t), \dots, N_k(t)) \\
N_2(t+1) &= G_2(N_1(t), N_2(t), \dots, N_k(t)) \\
&\vdots \\
N_k(t+1) &= G_k(N_1(t), N_2(t), \dots, N_k(t))
\end{aligned}
$$

Equilibrium

$\tilde{\mathbf{N}} = (\tilde{N}_1, \dots, \tilde{N}_k)$, such that

$$
\begin{aligned}
G_1(\tilde{N}_1, \dots, \tilde{N}_k) &= \tilde{N}_1 \\
&\vdots \\
G_k(\tilde{N}_1, \dots, \tilde{N}_k) &= \tilde{N}_k
\end{aligned}
$$

Stability analysis for discrete-time models

2. Write system of equations for the change over time of a small derivation from the equilibrium

Deviations from equilibrium

Define $n_i(t) = N_i(t) - \tilde{N}_i$.

Stability analysis for discrete-time models

2. Write system of equations for the change over time of a small derivation from the equilibrium

Deviations from equilibrium

Define $n_i(t) = N_i(t) - \tilde{N}_i$.

$$n_i(t+1) = G_i(N_1(t), \ldots, N_k(t)) - \tilde{N}_i$$

Stability analysis for discrete-time models

2. Write system of equations for the change over time of a small derivation from the equilibrium, and get a linear approximation of this system (Taylor series)

Deviations from equilibrium

Define $n_i(t) = N_i(t) - \tilde{N}_i$.

$$\begin{aligned} n_i(t+1) &= G_i(N_1(t), \dots, N_k(t)) - \tilde{N}_i \\ &\approx 0 + \left.\frac{\partial G_i}{\partial N_1}\right|_{\mathbf{N}(t)=\tilde{\mathbf{N}}} (N_1(t) - \tilde{N}_1) + \cdots + \left.\frac{\partial G_i}{\partial N_k}\right|_{\mathbf{N}(t)=\tilde{\mathbf{N}}} (N_k(t) - \tilde{N}_k). \end{aligned}$$

Stability analysis for discrete-time models

2. Write system of equations for the change over time of a small derivation from the equilibrium, and get a linear approximation of this system (Taylor series)

Deviations from equilibrium

Define $n_i(t) = N_i(t) - \tilde{N}_i$.

$$\begin{aligned} n_i(t+1) &= G_i(N_1(t), \dots, N_k(t)) - \tilde{N}_i \\ &\approx 0 + \left.\frac{\partial G_i}{\partial N_1}\right|_{\mathbf{N}(t)=\tilde{\mathbf{N}}} \underbrace{(N_1(t) - \tilde{N}_1)}_{n_1(t)} + \dots + \left.\frac{\partial G_i}{\partial N_k}\right|_{\mathbf{N}(t)=\tilde{\mathbf{N}}} \underbrace{(N_k(t) - \tilde{N}_k)}_{n_k(t)}. \end{aligned}$$

In matrix form:

$$\begin{pmatrix} n_1 \\ \vdots \\ n_k \end{pmatrix}(t+1) = \left.\begin{pmatrix} \frac{\partial G_1}{\partial N_1} & \dots & \frac{\partial G_1}{\partial N_k} \\ \vdots & \dots & \vdots \\ \frac{\partial G_k}{\partial N_1} & \dots & \frac{\partial G_k}{\partial N_k} \end{pmatrix}\right|_{\mathbf{N}=\tilde{\mathbf{N}}} \cdot \begin{pmatrix} n_1 \\ \vdots \\ n_k \end{pmatrix}(t)$$

Stability analysis for discrete-time models

2. Write system of equations for the change over time of a small derivation from the equilibrium, and get a linear approximation of this system (Taylor series)

Deviations from equilibrium

Define $n_i(t) = N_i(t) - \tilde{N}_i$.

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In matrix form:

$$\underbrace{\begin{pmatrix} n_1 \\ \vdots \\ n_k \end{pmatrix} (t+1)}_{\mathbf{n}(t+1)} = \underbrace{\left.\begin{pmatrix} \frac{\partial G_1}{\partial N_1} & \cdots & \frac{\partial G_1}{\partial N_k} \\ \vdots & \ldots & \vdots \\ \frac{\partial G_k}{\partial N_1} & \cdots & \frac{\partial G_k}{\partial N_k} \end{pmatrix}\right|_{\mathbf{N}=\tilde{\mathbf{N}}}}_{\mathbf{J}} \cdot \underbrace{\begin{pmatrix} n_1 \\ \vdots \\ n_k \end{pmatrix} (t)}_{\mathbf{n}(t)}$$

Stability analysis for discrete-time models (3)

3. Identify solutions of $\mathbf{n}(t+1) = \mathbf{J} \cdot \mathbf{n}(t)$

[?]

Stability analysis for discrete-time models (3)

3. Identify solutions of $\mathbf{n}(t+1) = \mathbf{J} \cdot \mathbf{n}(t)$

Solution:

$$\mathbf{n}(t) = c_1 \boldsymbol{\nu}_1 \lambda_1^t + c_2 \boldsymbol{\nu}_2 \lambda_2^t + \cdots + c_k \boldsymbol{\nu}_k \lambda_k^t,$$

with the c_i constants determined by the initial conditions, and $\boldsymbol{\nu}_{(i)}$ an eigenvector associated to the eigenvalue λ_i, i.e., $\mathbf{J} \cdot \boldsymbol{\nu}_{(i)} = \lambda_i \boldsymbol{\nu}_{(i)}$.

[?]

Stability analysis for discrete-time models (3)

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Leading eigenvalue: eigenvalue with the largest modulus

[?]

Stability analysis for discrete-time models (3)

3. Identify solutions of $\mathbf{n}(t+1) = \mathbf{J} \cdot \mathbf{n}(t)$

Solution:

$$\mathbf{n}(t) = c_1 \boldsymbol{\nu}_1 \lambda_1^t + c_2 \boldsymbol{\nu}_2 \lambda_2^t + \cdots + c_k \boldsymbol{\nu}_k \lambda_k^t,$$

with the c_i constants determined by the initial conditions, and $\boldsymbol{\nu}_{(i)}$ an eigenvector associated to the eigenvalue λ_i, i.e., $\mathbf{J} \cdot \boldsymbol{\nu}_{(i)} = \lambda_i \boldsymbol{\nu}_{(i)}$.

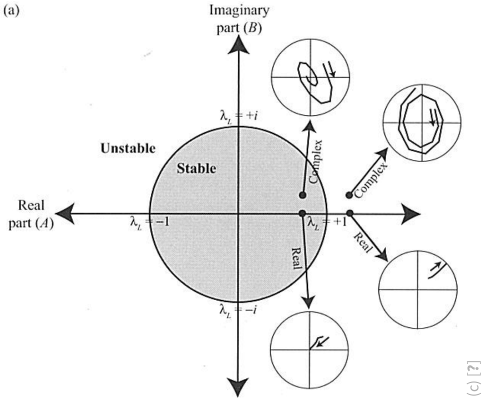
Leading eigenvalue: eigenvalue with the largest modulus
Modulus: for a complex number $\lambda = A + \imath B$,

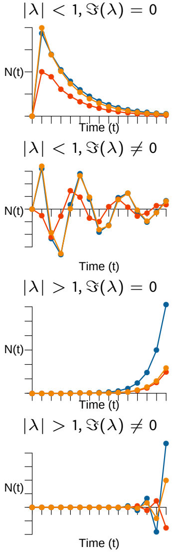
$$|\lambda| = \sqrt{A^2 + B^2}.$$

[**?**]

Stability analysis for discrete-time models (4)

4 Inspect the eigenvalues of **J**





Back