Victimless robbery

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Group 7

IX Southern-Summer Schools on Mathematical Biology (SSSBM 2020)

January 12 – 18, 2020

São Paulo, Brazil

ITCP-SAIFR/IFT-Unesp

Section 1

Introduction

Once upon a time there was a beautiful garden...

Mutualism: positive plant-animal interaction

Plants give nectar to animals

Animals exchange pollen among flowers - reproduction!

But... peace was disturbed!

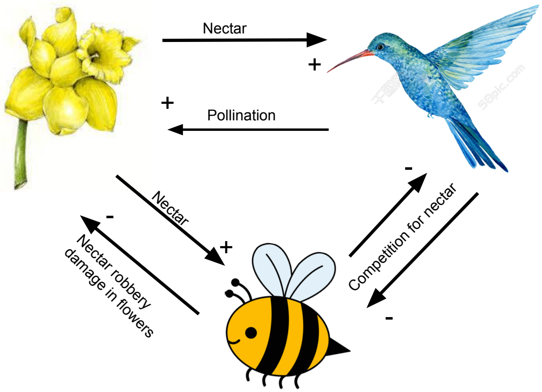
A series of nectar robbery started to scare the community!

Damage in the flowers

The suspect

Bumblebee

Trapped in Darwin's web



How does the robber affect the mutualism?

- Is the nectar robber only a bad guy?
- New studies found that robbers can increase reproduction success of the plants!!
- But how?

The mechanisms for a positive effect of robbers

- Robbers compete for nectar with pollinators
- Less nectar available - pollinators have to search for more flowers
- Pollinators visit more flowers and increase reproduction success of plants

Objectives

- Construct a model to describe plant-pollinator-robber system
- Analyse how robbers presence may affects positively pollination

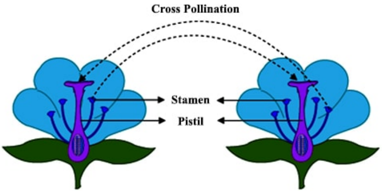
Section 2

Dynamic model

Our delicate system

Plants/Flowers

- Pollination is specialized - done by one species of animal.
- No auto pollination!
- No costs to produce nectar
- The production of nectar is proportional to the number of flowers



flower biological processes

Our beautiful system

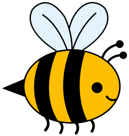
Pollinators - hummingbirds

- Specialized in the species of plant.
- Population growth by nectar consumption
- Increases flower visit rates (and pollination) when the amount of nectar per flower is low.
- No cost for searching for more flowers.

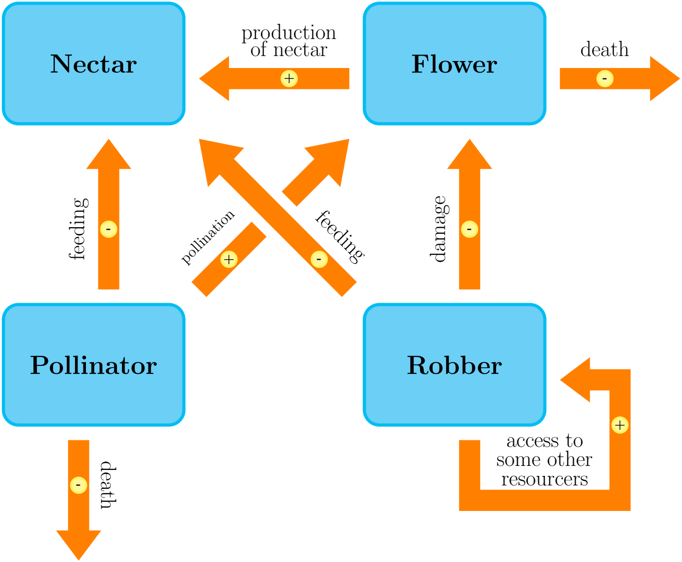
Our cute system

Robbers - bumblebees

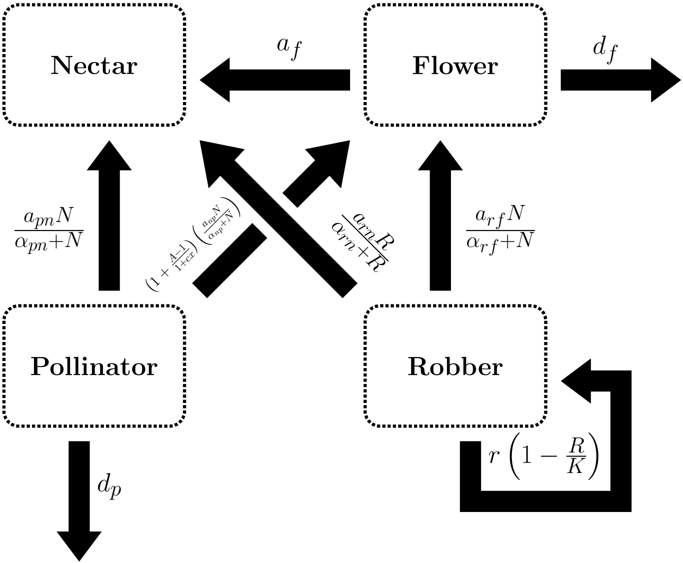
- Generalists.
- Competing by interference with the pollinators for nectar.
- Opens a hole in the flower - damage/kill the flower.



Our -crazy- system



Nectar-Flower-Pollinator-Robber dynamics



Associated dynamic system

$$\frac{dF}{dt} = \left(\left(1+\frac{A-1}{1+cN}\right)\left(\frac{a_{pf}P}{1+\beta_f F+\beta_p P}\right) - \frac{a_{fr}R}{\alpha_{fr}+R} - d_f\right)F$$

$$\frac{dN}{dt} = a_f F - \left(\frac{a_{rn}R}{\alpha_{rn}+N} + \frac{a_{pn}P}{\alpha_{pn}+N}\right)N$$

$$\frac{dP}{dt} = \left(\frac{a_{pn}N}{\alpha_{pn}+N} - d_p\right)P$$

$$\frac{dR}{dt} = \left(r\left(1-\frac{R}{K}\right) + \frac{a_{nr}N}{\alpha_{rn}+N}\right)R \quad + \text{ initial conditions}$$

N - Volume of nectar
F - Population of flowers
P - Population of pollinators
R - Population of robbers

A - low-nectar amplification of pollination behavior
c - speed of pollination behavior transition

a_{pf} - pollination rate
β_f - visitation rate of the pollinators
β_p - saturation rate of the pollinators
a_{fr} - damage rate by robber
α_{fr} - half saturation for robber attacks
d_r - death rate of the flowers
a_f - production rate of nectar per flower
rn - inverse of handling time of robbers

α_{rn} - half saturation of nectar for robbers
a_{pn} - inverse of handling time of pollinators
α_{pn} - half saturation of nectar for pollinators
d_p - death rate of pollinators
r - growth rate of robbers
K - carrying capacity of the robbers
a_{nr} - robber growth rate boost
a_{np} - pollinator birth-rate

Parameters choice

Looking for information in the vast literature:

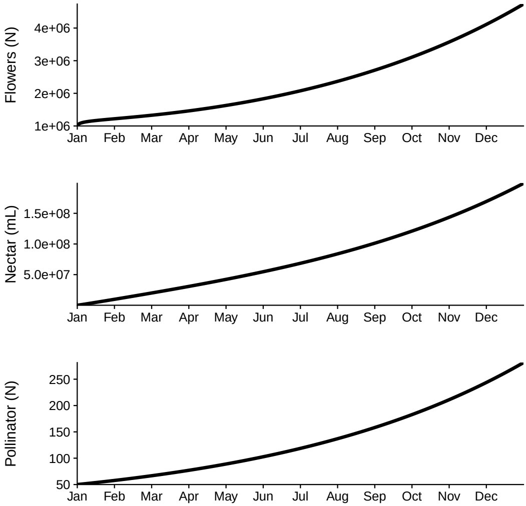
- Amount of nectar per flower: 0.5 μL
- Pollination rate for hummingbirds: 2000 visits/day and "handling time" as 20 seconds on average
- Robbery rate for bumblebees: 1000 visits/day, effective "handling time" of 2000 seconds on average

+ some reasonable guesses about death rates, robbers intrinsic growth rates and carrying capacity

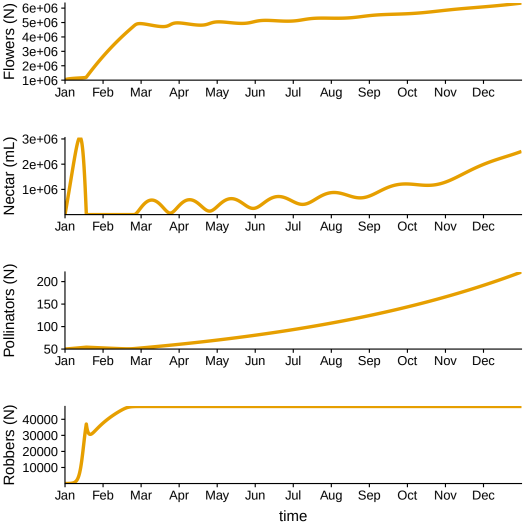
Section 3

Results

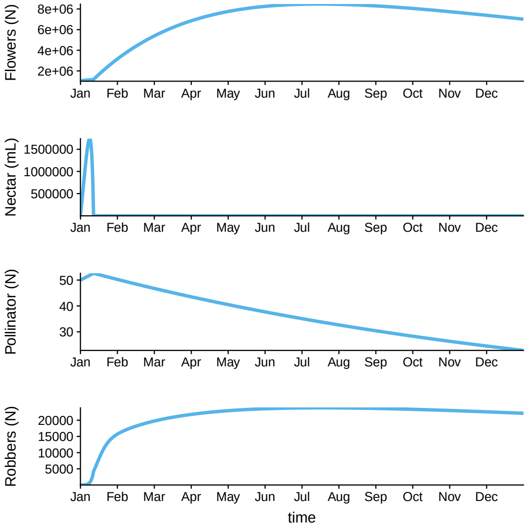
Pop. dynamics WITHOUT robbers



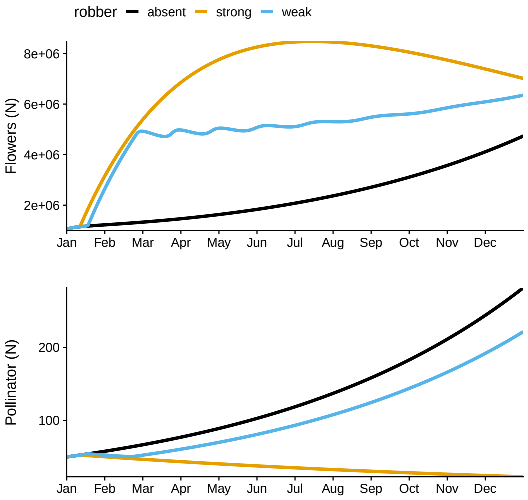
Pop. dynamics WITH WEAK robbers



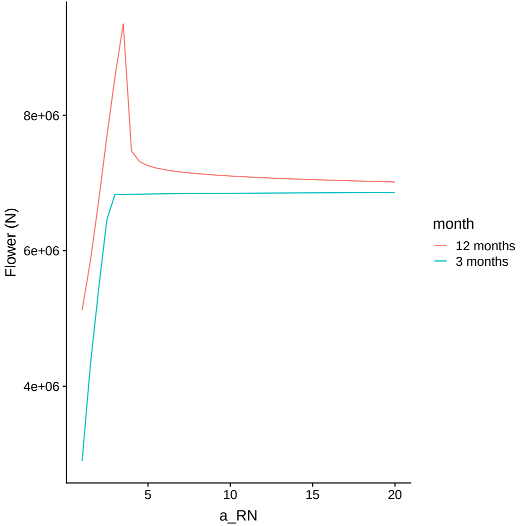
Pop. dynamics WITH STRONG robbers



Comparing Results



Bifurcation diagram for consumption rate for robbers



Section 4

Discussion

Discussion

Are nectar robbers real bad for the mutualism?

- It depends on how better competitor they are:
- If strong competitors, bad guy!)
- If weak competitors, maybe they may coexist
- Weak robbers may help in the stabilization of the population growth of plants, but not so strongly because it didn't impede pollinators growth

Future directions

Improvements in the model

- Include carrying capacity for the plants and pollinators
- Include costs for the flight time looking for nectar in pollinators and robbers - it will control it's growth rate
- Play with specialization in plants-pollinator systems

Lessons from Nature

- Mutualism is a very hard species interaction to model!
- Robbers are not always the bad guys in the story!
- They may also increase the reproduction of plants by forcing lazy pollinators to work!

Acknowledgements

Monitors:

Rafael Pamela Matheus

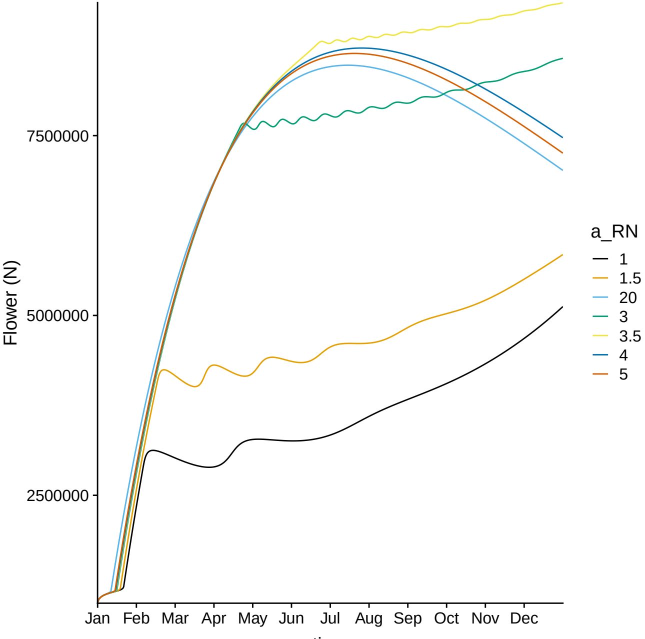
Financial support:

ICTP SAIFR

That's all folks!

Questions?!

Thanks for your attention!


Flower (N)
7500000
5000000
2500000
Jan
Feb
Mar
Apr
May
Jun
Jul
Aug
Sep
Oct
Nov
Dec
a_RN
1
1.5
20
3
3.5
4
5

$$\frac{dF}{dt} = \left(\left(1+\frac{A-1}{1+cN}\right)\left(\frac{a_{pf}P}{1+\beta_f F+\beta_p P}\right)-\frac{a_{fr}R}{\alpha_{fr}+R}-d_f\right)F$$

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Initial conditions

Initial condition	What it is	Value
F_0	Initial population of flowers	10^6
N_0	Initial amount of nectar	$(5\ \mu\mathrm{L}) \cdot F_0$
P_0	Initial population of pollinators (hummingbirds)	50
R_0	Initial population of robbers (bumblebees)	10

Parameter values

Parameter	Info	Coexistence	Extermination
A	Low nectar amplification of pollination	10	10
c	Speed of pollination behavior transition	10^{-5}	10^{-5}
a_{pf}	Pollination rate per flower	10^{-2}	10^{-2}
β_f	Visitation rate of pollinators per flower	10^{-3}	10^{-3}
β_p	Saturation rate of pollination per pollinator	$2 \cdot 10^{-3}$	$2 \cdot 10^{-3}$
a_{fr}	Damage rate due to robber per flower	10^{-8}/h	10^{-6}/h
α_{fr}	Robber attack half-saturation	8000	8000
d_f	Per capita flower death rate	1%/day	1%/day
a_f	Nectar production rate per flower	20 μL/day	20 μL/day
$a_{rn} = h_r^{-1}$	Inverse of the robber handling time for nectar	1.8 h^{-1}	18 h^{-1}
α_{rn}	Nectar half-saturation for robbers	0.05 μL	0.05 μL
$a_{pn} = h_p^{-1}$	Inverse of the pollinator handling time for nectar	180 h^{-1}	180 h^{-1}
α_{pn}	Nectar half-saturation for pollinators	20 μL	20 μL
d_p	Per capita pollinator death rate	1%/day	1%/day
r	Robber growth rate (in absence of nectar)	$4 \cdot 10^{-3}$/h	$4 \cdot 10^{-3}$/h
K	Robber carrying capacity (in absence of nectar)	8000	8000
a_{nr}	Robber per capita growth rate boost due to nectar availability	$2 \cdot 10^{-2}$/h	$2 \cdot 10^{-2}$/h
a_{np}	Pollinator per capita growth rate due to nectar availability	$3 \cdot 10^{-4}$/h	$3 \cdot 10^{-4}$/h