Hawkmoths: pollinators that are herbivores

Julia Oshima, Laila Kazimierski, Lucas S. Simões, Lucas Nascimento, Virgilio Vazquez Hipólito, Yazmín H. Zurita-Gutiérrez

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ANNALS OF BOTANY Founded 1887

Reproductive biology of *Datura wrightii*: the benefits of a herbivorous pollinator

Judith L. Bronstein[1,*], Travis Huxman[1], Brianna Horvath[2], Michael Farabee[3] and Goggy Davidowitz[1]
[1]*Department of Ecology and Evolutionary Biology, University of Arizona, Tucson, AZ, USA,* [2]*Jefferson Medical College, Thomas Jefferson University, Philadelphia, PA, USA and* [3]*Division of Science, Estrella Community College, Avondale, AZ, USA*

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The System

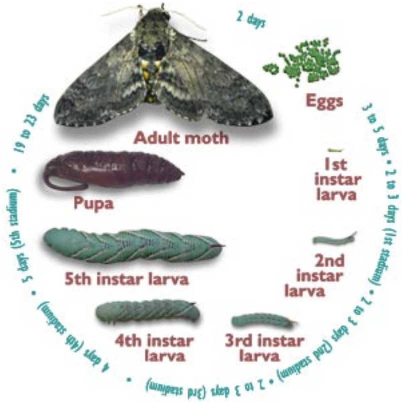


Figure: Hawkmoth Lifecycle

Photo: http://insected.arizona.edu/manduca

The System

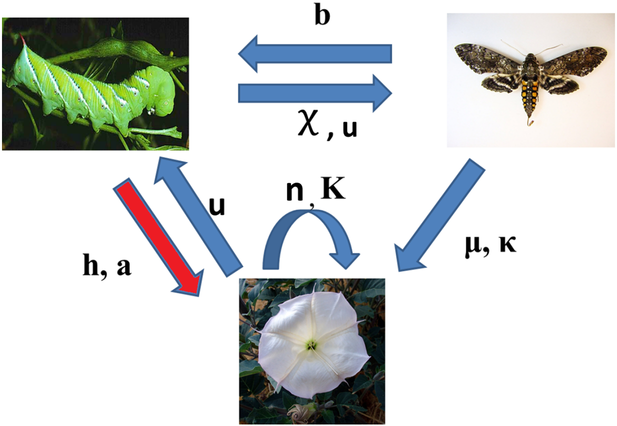


Figure: Model Diagram

Biological Facts

- *Datura wrightii* (P) is highly self-compatible
- Plant leaves are the only food of *Manduca sexta larvae* (M_L)
- But *M. sexta* adults (M_A) don't depend exclusively on P
- Pollinated flowers by M_A produce more fruit and seeds
- The floral visitation component of the moth-plant interaction is mutualistic

Objectives

Identify the key biological aspects of the system.

Create a minimal model that describes the population dynamics of the system P-M_L-M_A.

Can this model predict coexistence of a plant and a pollinator that is also its herbivore?

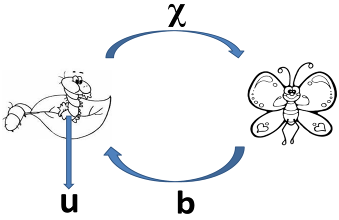
System of Equations

$$
\begin{aligned}
\frac{dP}{dt} &= \left(n + \frac{\kappa M_A}{1+\mu M_A}\right) P \left(1 - \frac{P}{K}\right) - \frac{a M_L P}{1 + ahP} \\
\frac{dM_L}{dt} &= b M_A - m_L M_L - \frac{\chi M_L P}{1 + uP} \\
\frac{dM_A}{dt} &= \frac{\chi M_L P}{1 + uP} - m_A M_A
\end{aligned}
$$

System of Equations

Setting some parameters:

$$\begin{aligned}\frac{dP}{dt} &= \left(1+\frac{3M_A}{1+M_A}\right)P\left(1-P\right)-\frac{M_LP}{1+0.3P}\\ \frac{dM_L}{dt} &= bM_A-0.2M_L-\frac{\chi M_LP}{1+uP}\\ \frac{dM_A}{dt} &= \frac{\chi M_LP}{1+uP}-0.2M_A\end{aligned}$$



Analytical Analysis

Now we search for equilibria:

$$\frac{dP}{dt} = \frac{dM_A}{dt} = \frac{dM_L}{dt} = 0$$

$$P^* = \frac{1}{\frac{\chi(b-m_A)}{m_L m_A} - u}$$

$$M_L^* = \left(n + \frac{3M_A^*}{1+M_A^*}\right)(1+hP^*)(1-P^*)$$

$$M_A^* = \frac{\chi P^* M_L^*}{m_A(1+uP^*)}$$

Analytical Analysis

P solution:
Gives a condition in order to get a biologically relevant equilibrium:

$$P^* = \frac{1}{\frac{\chi(b-m_A)}{m_L m_A} - u} \Rightarrow \frac{u}{\chi} < \frac{b-m_A}{m_L m_A}$$

ANALYTICAL ANALYSIS

Figure: M_L Solution

Analytical Analysis

```
In [2]: s[1][0]
Out[2]: 25.0*x**2*(5.0*b - 1.0)*(sqrt(152587890625.0*b**8*x**8 - 48828125000.0*b**7*u*x**7 - 366210937500.0*b**7*x**8 - 4882812500.0*b
**6*h*x**7 + 6835937500.0*b**6*u**2*x**6 + 97656250000.0*b**6*u*x**7 + 439453125000.0*b**6*x**8 + 4882812500.0*b**6*x**7 + 976562500.0
*b**5*h*u*x**6 + 13671875000.0*b**5*h*x**7 + 195312500.0*b**5*h*x**6 - 546875000.0*b**5*u**3*x**5 - 11132812500.0*b**5*u**2*x**6 - 917
96875000.0*b**5*u*x**7 - 976562500.0*b**5*u*x**6 - 288085937500.0*b**5*x**8 - 13671875000.0*b**5*x**7 + 156250000.0*b**4*h**2*x**6 - 7
8125000.0*b**4*h*u**2*x**5 - 1914062500.0*b**4*h*u*x**6 - 31250000.0*b**4*h*u*x**5 - 10742187500.0*b**4*h*x**7 - 820312500.0*b**4*h*x*
*6 + 27343750.0*b**4*u**4*x**4 + 703125000.0*b**4*u**3*x**5 + 7968750000.0*b**4*u**2*x**6 + 78125000.0*b**4*u**2*x**5 + 46875000000.0*
b**4*u*x**7 + 1914062500.0*b**4*u*x**6 + 109863281250.0*b**4*x**8 + 10742187500.0*b**4*x**7 + 156250000.0*b**4*x**6 - 12500000.0*b**3*
h**2*u*x**5 - 125000000.0*b**3*h**2*x**6 - 12500000.0*b**3*h**2*x**5 + 3125000.0*b**3*h*u**3*x**4 + 100000000.0*b**3*h*u**2*x**5 + 187
5000.0*b**3*h*u**2*x**4 + 1140625000.0*b**3*h*u*x**6 + 75000000.0*b**3*h*u*x**5 + 3906250000.0*b**3*h*x**7 + 578125000.0*b**3*h*x**6 +
 12500000.0*b**3*h*x**5 - 875000.0*b**3*u**5*x**3 - 26562500.0*b**3*u**4*x**4 - 368750000.0*b**3*u**3*x**5 - 3125000.0*b**3*u**3*x**4
- 3015625000.0*b**3*u**2*x**6 - 100000000.0*b**3*u**2*x**5 - 13671875000.0*b**3*u*x**7 - 1140625000.0*b**3*u*x**6 - 12500000.0*b**3*u*
x**5 - 25195312500.0*b**3*x**8 - 3906250000.0*b**3*x**7 - 125000000.0*b**3*x**6 + 250000.0*b**2*h**2*u**2*x**4 + 7500000.0*b**2*h**2*u
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*4*x**3 - 2375000.0*b**2*h*u**3*x**4 - 50000.0*b**2*h*u**3*x**3 - 41250000.0*b**2*h*u**2*x**5 - 2125000.0*b**2*h*u**2*x**4 - 303125000
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 100000.0*b*h**2*u**2*x**4 - 1500000.0*b*h**2*u*x**5 - 200000.0*b*h**2*u*x**4 - 5000000.0*b*h**2*x**6 - 1500000.0*b*h**2*x**5 - 100000
.0*b*h**2*x**4 + 500.0*b*h*u**5*x**2 + 25000.0*b*h*u**4*x**3 + 500.0*b*h*u**4*x**2 + 575000.0*b*h*u**3*x**4 + 20000.0*b*h*u**3*x**3 +
7000000.0*b*h*u**2*x**5 + 625000.0*b*h*u**2*x**4 + 37812500.0*b*h*u*x**6 + 7000000.0*b*h*u*x**5 + 200000.0*b*h*u*x**4 + 71875000.0*b*h
*x**7 + 21562500.0*b*h*x**6 + 1500000.0*b*h*x**5 - 200.0*b*u**7*x - 7500.0*b*u**6*x**2 - 135000.0*b*u**5*x**3 - 500.0*b*u**5*x**2 - 15
37500.0*b*u**4*x**4 - 25000.0*b*u**4*x**3 - 12375000.0*b*u**3*x**5 - 575000.0*b*u**3*x**4 - 66562500.0*b*u**2*x**6 - 7000000.0*b*u**2*
x**5 - 100000.0*b*u**2*x**4 - 203125000.0*b*u*x**7 - 37812500.0*b*u*x**6 - 1500000.0*b*u*x**5 - 257812500.0*b*x**8 - 71875000.0*b*x**7
 - 5000000.0*b*x**6 + 10000.0*h**2*u**2*x**4 + 100000.0*h**2*u*x**5 + 20000.0*h**2*u*x**4 + 250000.0*h**2*x**6 + 100000.0*h**2*x**5 +
10000.0*h**2*x**4 - 100.0*h*u**5*x**2 - 2500.0*h*u**4*x**3 - 100.0*h*u**4*x**2 - 45000.0*h*u**3*x**4 - 2000.0*h*u**3*x**3 - 425000.0*h
*u**2*x**5 - 55000.0*h*u**2*x**4 - 1812500.0*h*u*x**6 - 450000.0*h*u*x**5 - 20000.0*h*u*x**4 - 2812500.0*h*x**7 - 1062500.0*h*x**6 - 1
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25000.0*u**3*x**5 + 45000.0*u**3*x**4 + 2875000.0*u**2*x**6 + 425000.0*u**2*x**5 + 10000.0*u**2*x**4 + 7500000.0*u*x**7 + 1812500.0*u*
x**6 + 100000.0*u*x**5 + 8203125.0*x**8 + 2812500.0*x**7 + 250000.0*x**6)/(156250.0*b**3*x**4 - 12500.0*b**2*u*x**3 - 93750.0*b**2*x**
4 + 250.0*b*u**2*x**2 + 5000.0*b*u*x**3 + 18750.0*b*x**4 - 50.0*u**2*x**2 - 500.0*u*x**3 - 1250.0*x**4) - 0.02*(390625.0*b**4*x**4 - 6
2500.0*b**3*u*x**3 - 625000.0*b**3*x**4 - 12500.0*b**2*h*x**3 + 3750.0*b**2*u**2*x**2 + 62500.0*b**2*u*x**3 + 281250.0*b**2*x**4 + 125
00.0*b**2*x**3 + 500.0*b*h*u*x**2 + 5000.0*b*h*x**3 + 500.0*b*h*x**2 - 100.0*b*u**3*x - 2000.0*b*u**2*x**2 - 17500.0*b*u*x**3 - 500.0*
b*u*x**2 - 50000.0*b*x**4 - 5000.0*b*x**3 - 100.0*h*u*x**2 - 500.0*h*x**3 - 100.0*h*x**2 + u**4 + 20.0*u**3*x + 250.0*u**2*x**2 + 1500
.0*u*x**3 + 100.0*u*x**2 + 3125.0*x**4 + 500.0*x**3)/(x**2*(5.0*b - 1.0)*(25.0*b*x - u - 5.0*x)**2))/(25.0*b*x - u - 5.0*x)**2
```

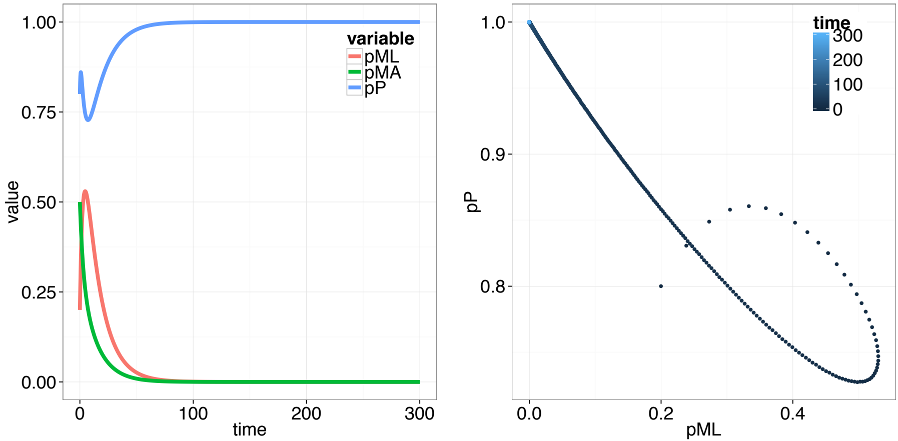
Figure: M_A Solution

Analytical Analysis

$$J = \begin{pmatrix} -P^*(1 + \frac{3M_A^*}{1+M_A^*} + \frac{0.3M_L^*}{(1+hP^*)^2}) & \frac{-P^*}{1+hP^*} & \frac{3P^*(1-P^*)}{(1+M_A^*)^2} \\ \frac{-\chi M_L^*}{(1+uP^*)^2} & -0.2 - \frac{\chi P^*}{1+uP^*} & b \\ \frac{\chi M_L^*}{(1+uP^*)^2} & \frac{\chi P^*}{1+uP^*} & -0.2 \end{pmatrix}$$

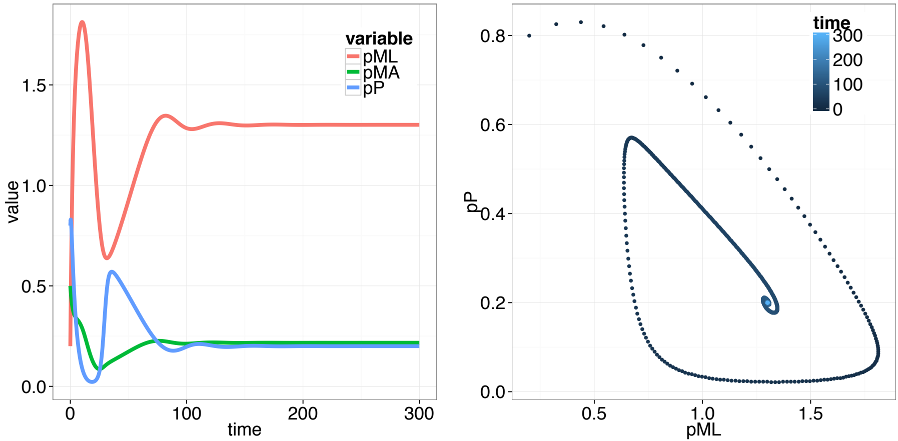
Graphics

Setting $b = 0.5, u = 10$ (real part of eigenvalues negative):



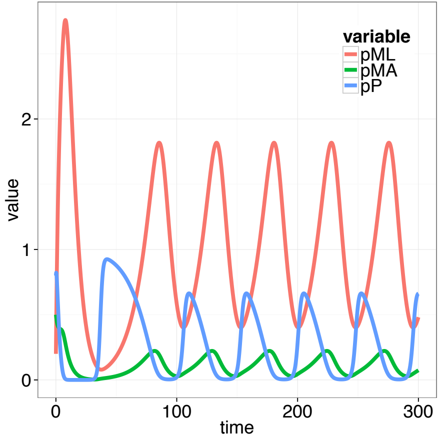
Graphics

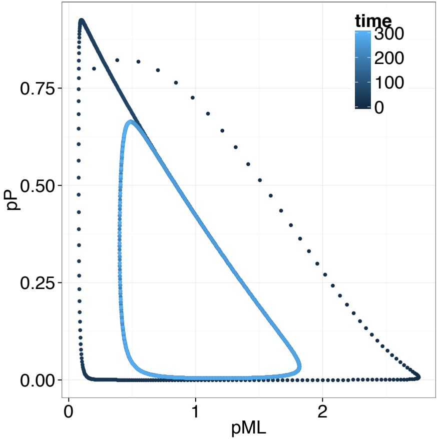
Setting $b = 1.4, u = 10$ (real part of eigenvalues negative):



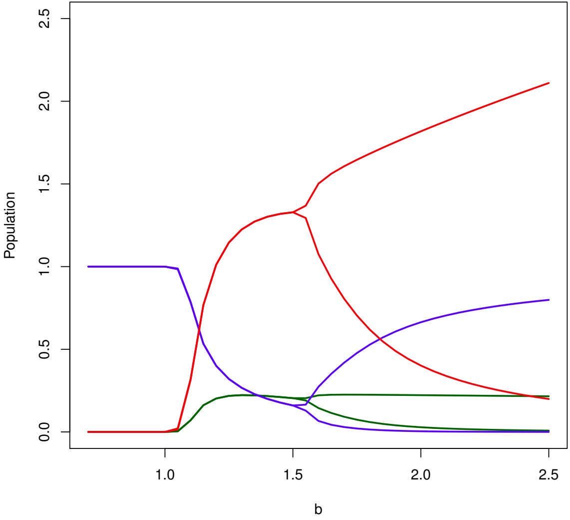
GRAPHICS

Setting $b = 2.0, u = 10$ (at least one eigenvalue positive):



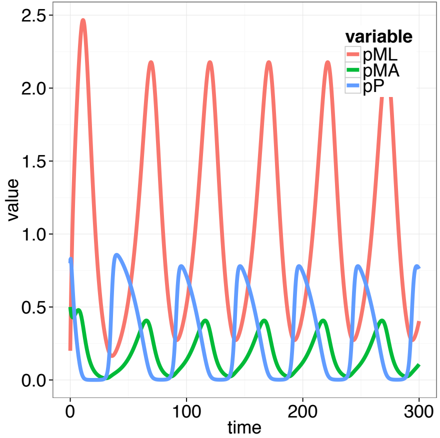


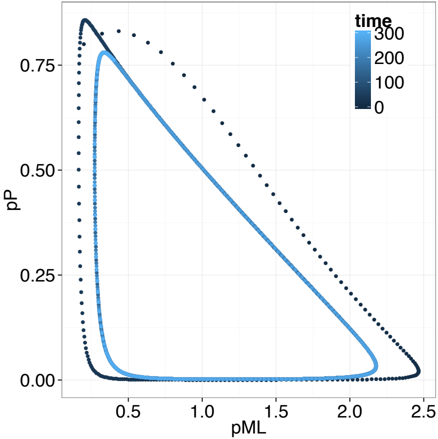
BIFURCATION DIAGRAM FOR b



GRAPHICS

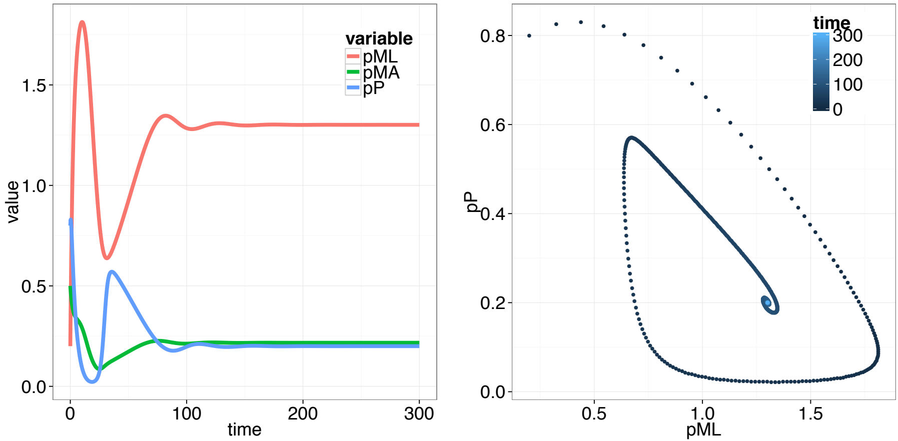
Setting $b = 1.4, u = 5$ (at least one eigenvalue positive):





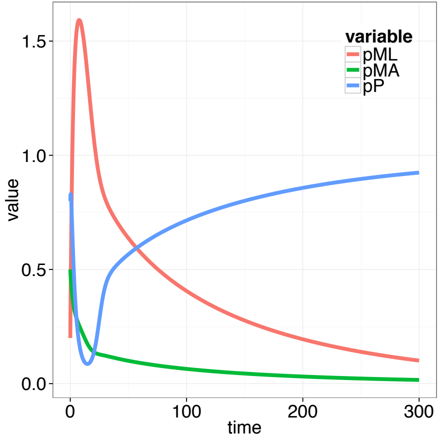
Graphics

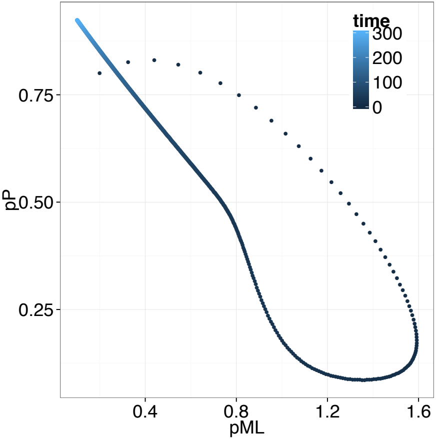
Setting $b = 1.4, u = 10$ (real part of eigenvalues negative):



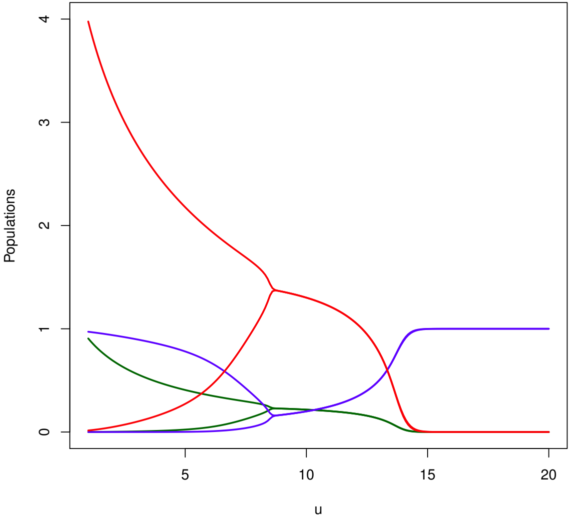
GRAPHICS

Setting $b = 1.4, u = 14$ (real part of eigenvalues negative):





Bifurcation Diagram for u



Conclusions

- This model can describe the population dynamics of the system P-M_L-M_A
- We identified three parameters (b, u, χ) that drive diferent dynamics of the system:
  - Extintion of the moth
  - Coexistence
  - Oscillations

Future steps

- Evaluate the effects of K varying with M_A instead of being a fixed value
- Rescale the variables
- Make an exhaustive exploration of parameter values' variation

Bibliography

- Bronstein, J.L., T.H. Huxman, and G. Davidowitz (2009). "Reproductive biology of Datura wrightii: the benefits of associating with an herbivorous pollinator", Annals of Botany 103: 1435-1443.
- Bronstein, J.L., T.E. Huxman, and G. Davidowitz. "Ecological Communities", Chapter 4: Plant-mediated effects linking herbivory and pollination, 1st ed. Cambridge: Cambridge University Press, 2007. 75-103. Cambridge Books Online http://dx.doi.org/10.1017/CBO9780511542701.005.
- http://eebweb.arizona.edu/faculty/bronstein/Bronstein_Lab/PROJECTS.html

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You for your attention