A mathematical model for the eradication of Guinea worm disease

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Outline

- Biology/epidemiology of Guinea worm disease

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- Mathematical model

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- Impulsive differential equations

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- Implications.

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- Europeans first saw the disease on the Guinea coast of West Africa in the 17th century.

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- It resides here for about a year.

The Guinea Worm

Transmission

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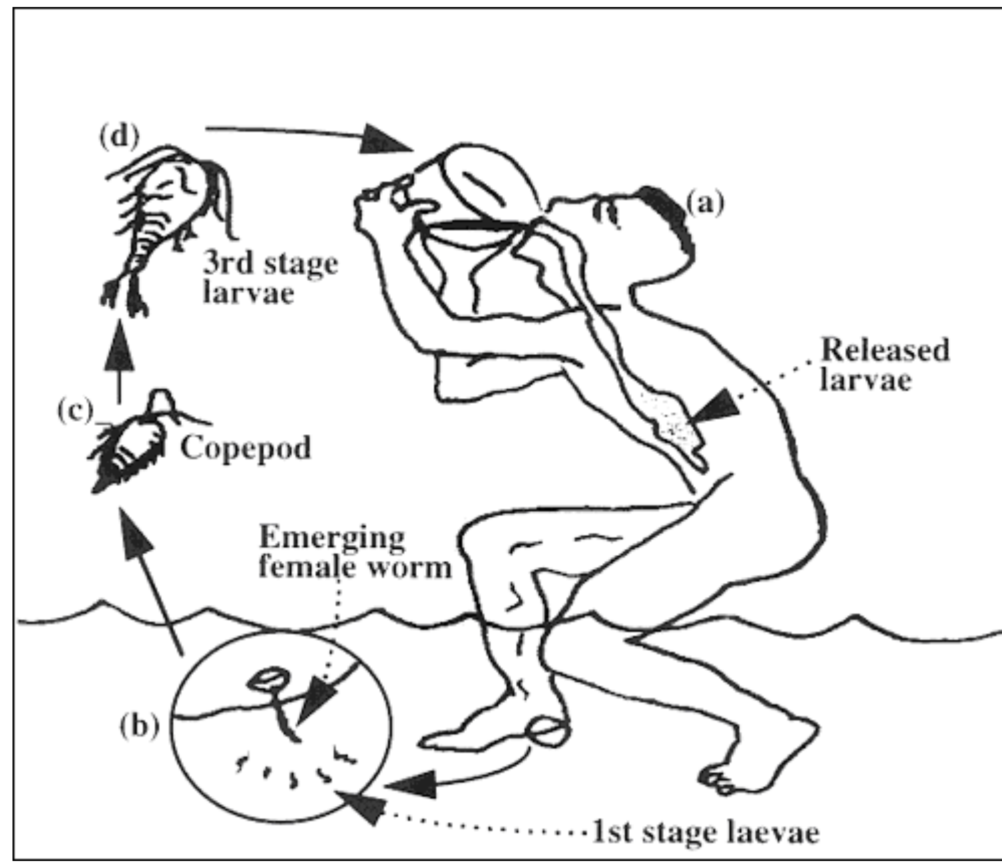

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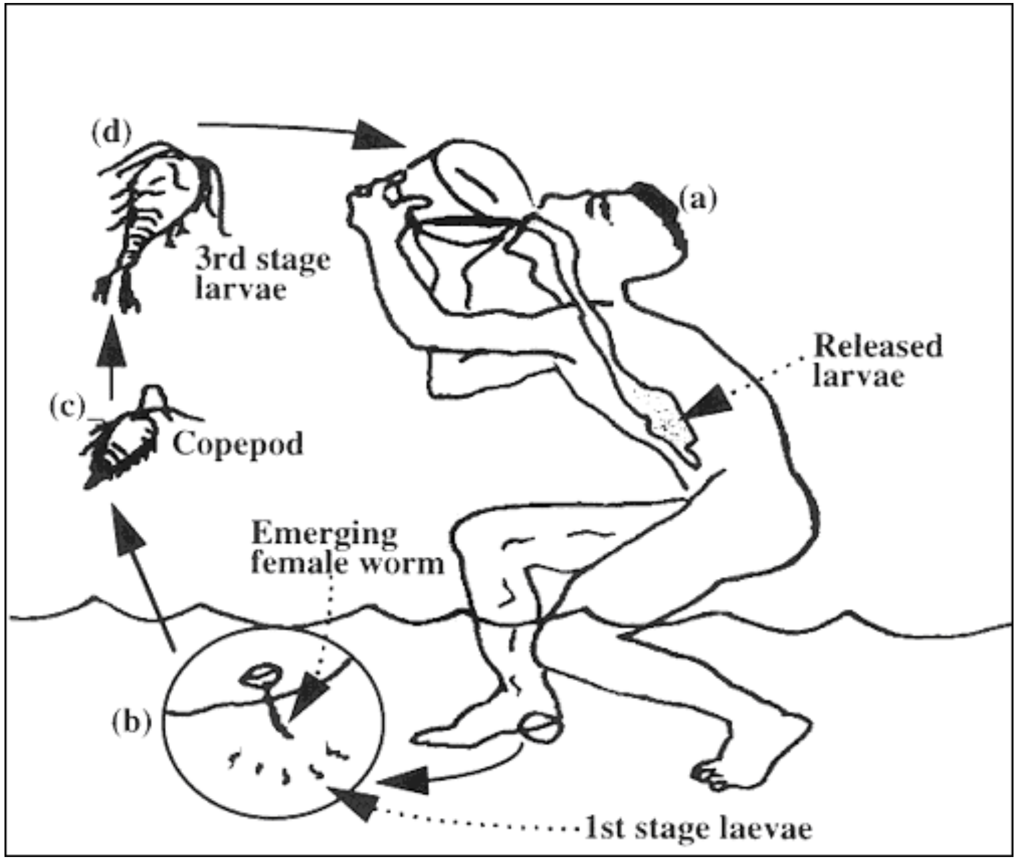

Transmission

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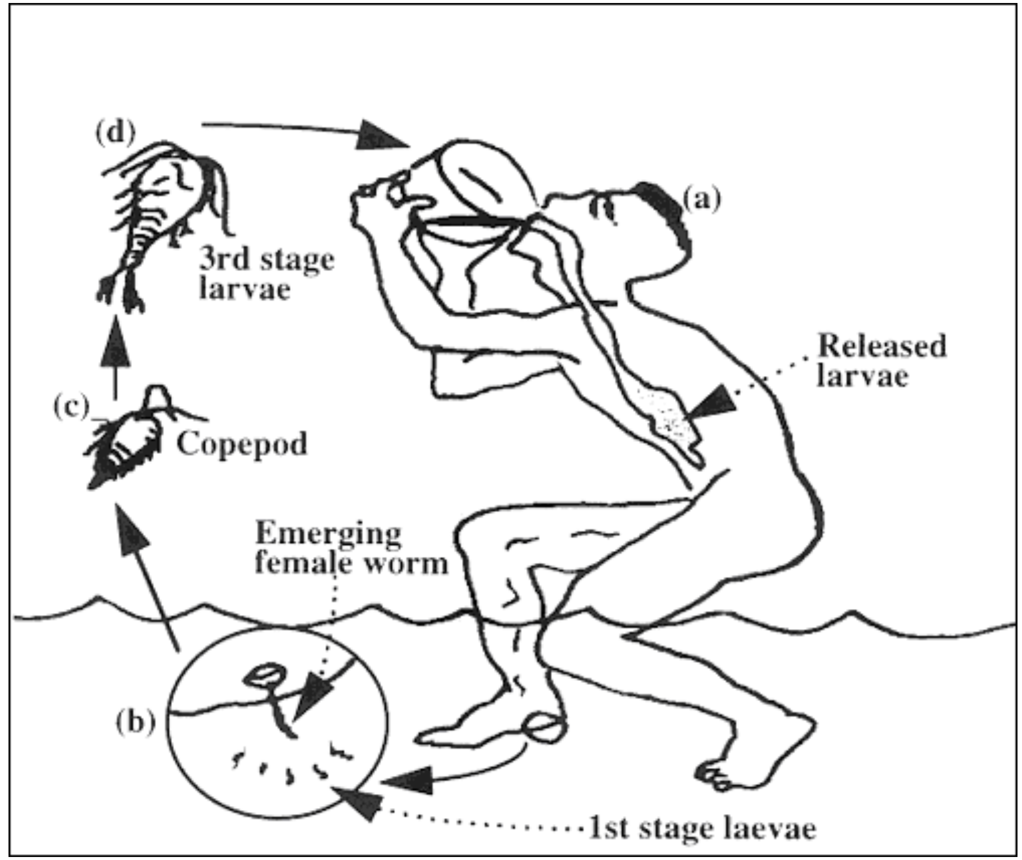
- The host places the infected limb in water
- At this point, the worm ejects hundreds of thousands of larvae, restarting the cycle.



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- This takes up to two months.

Burden of infection

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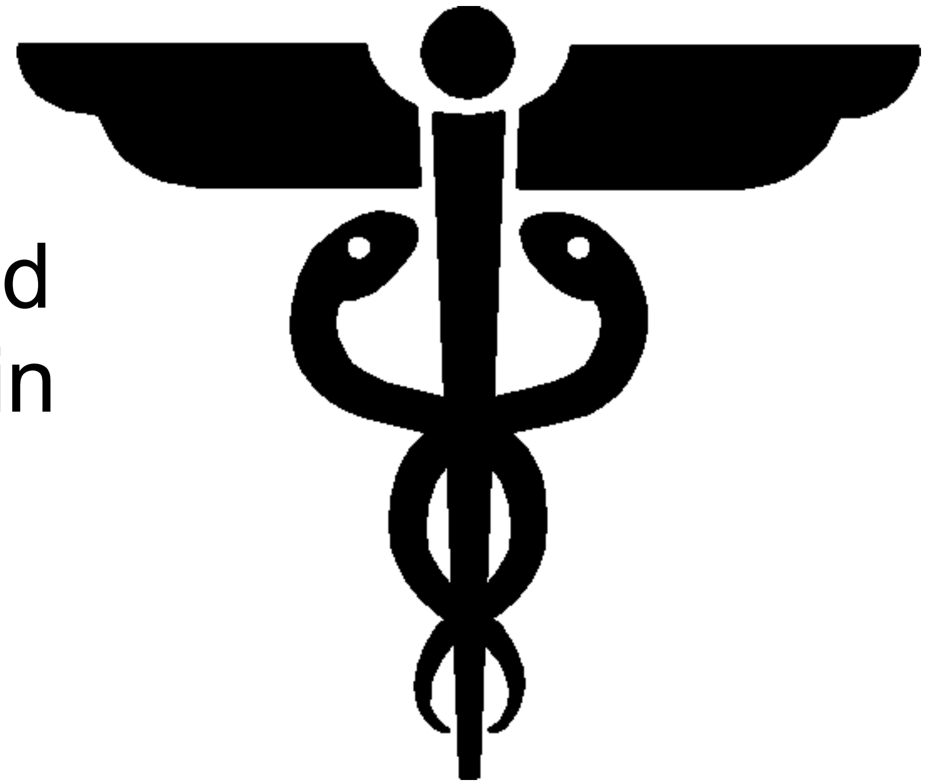

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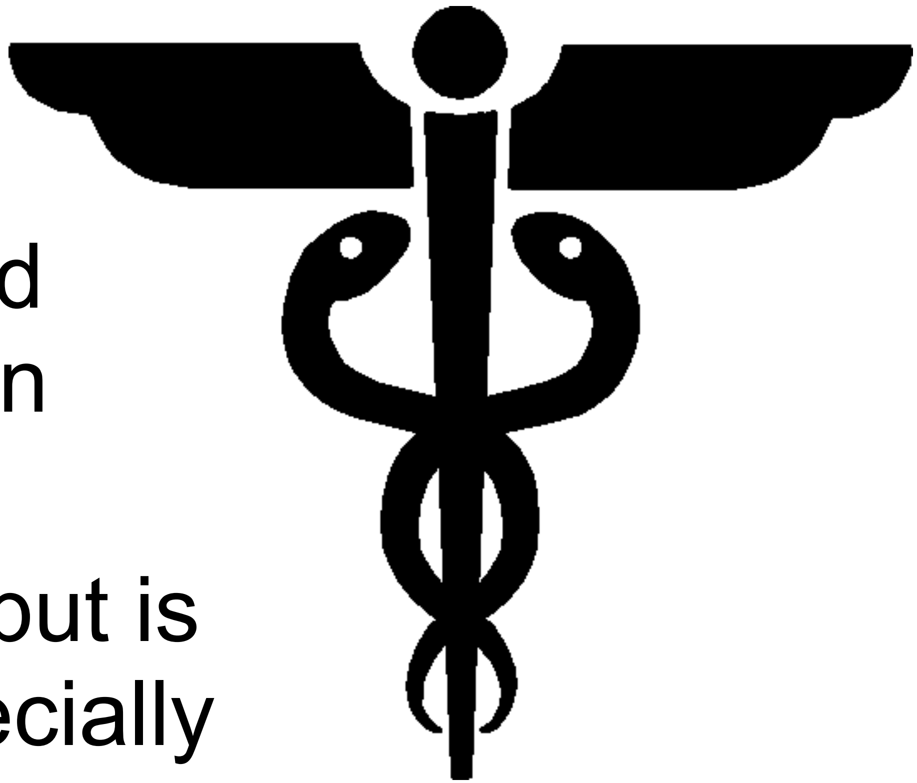

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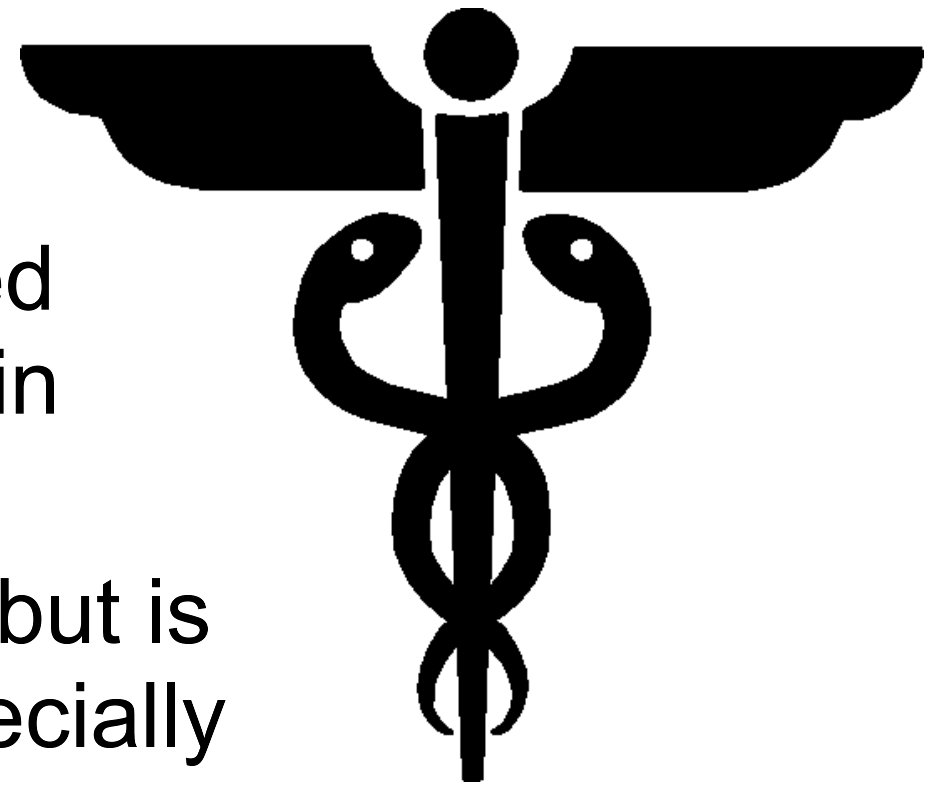

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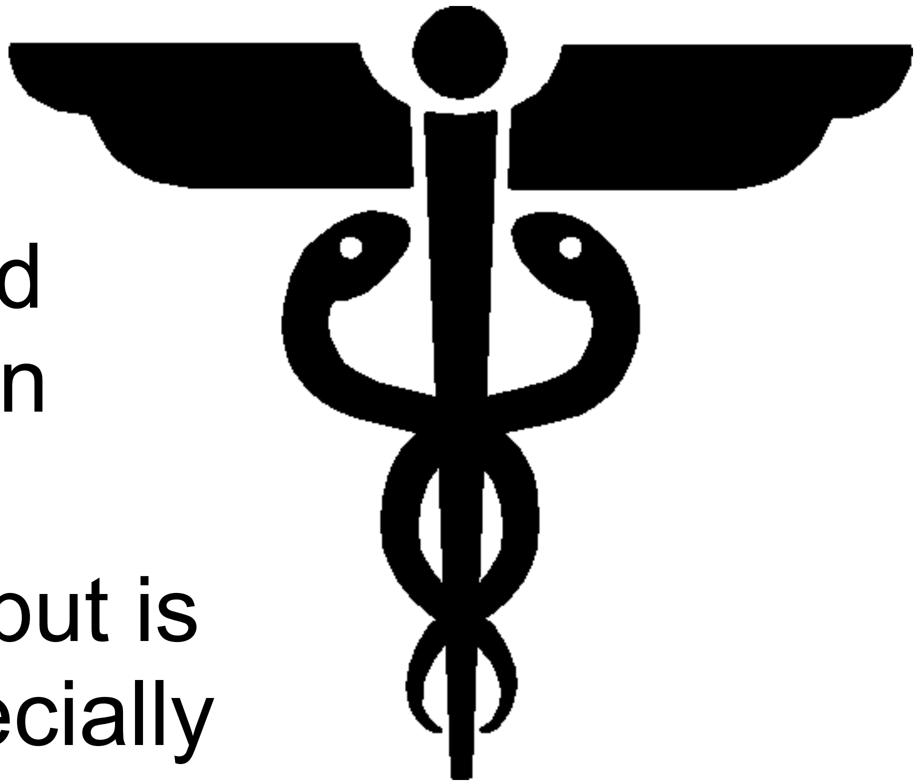

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Geography

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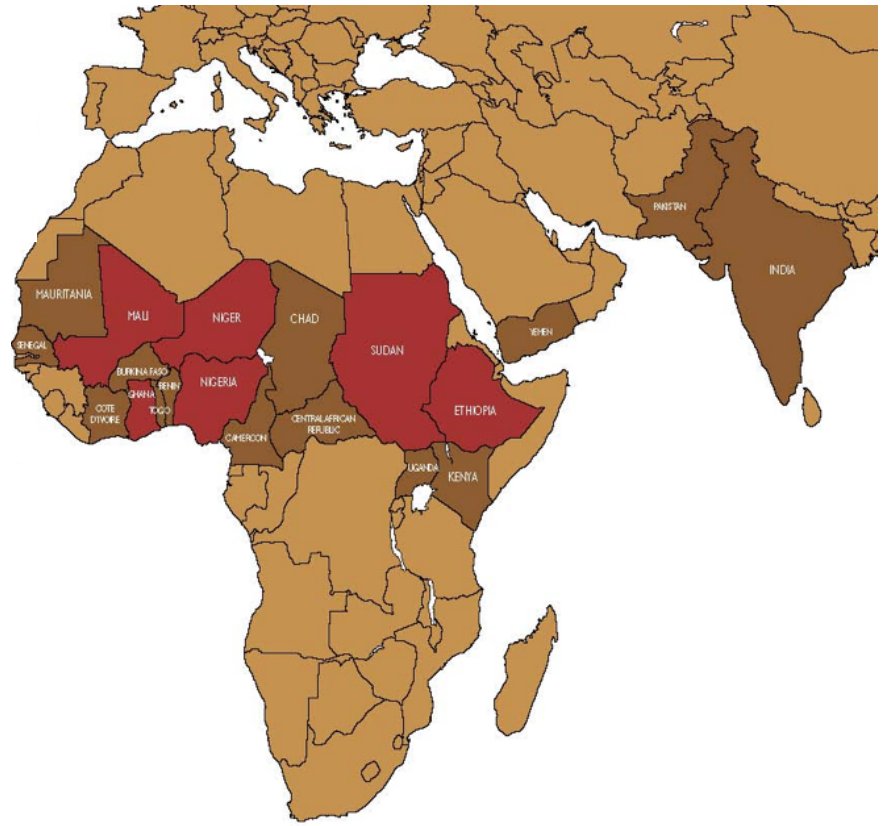

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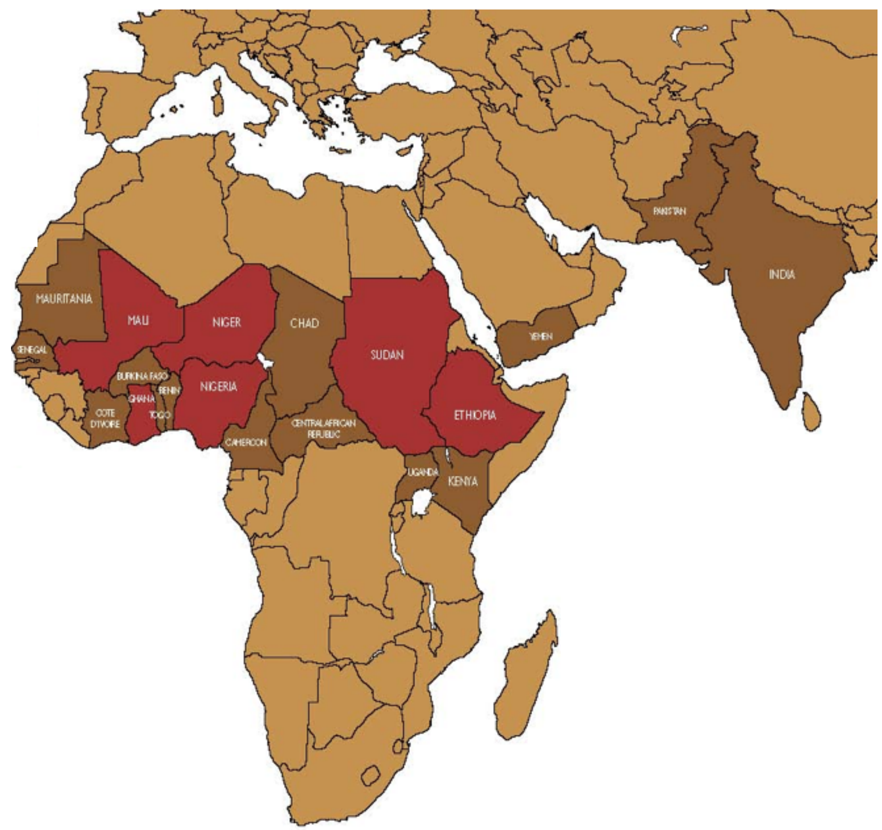

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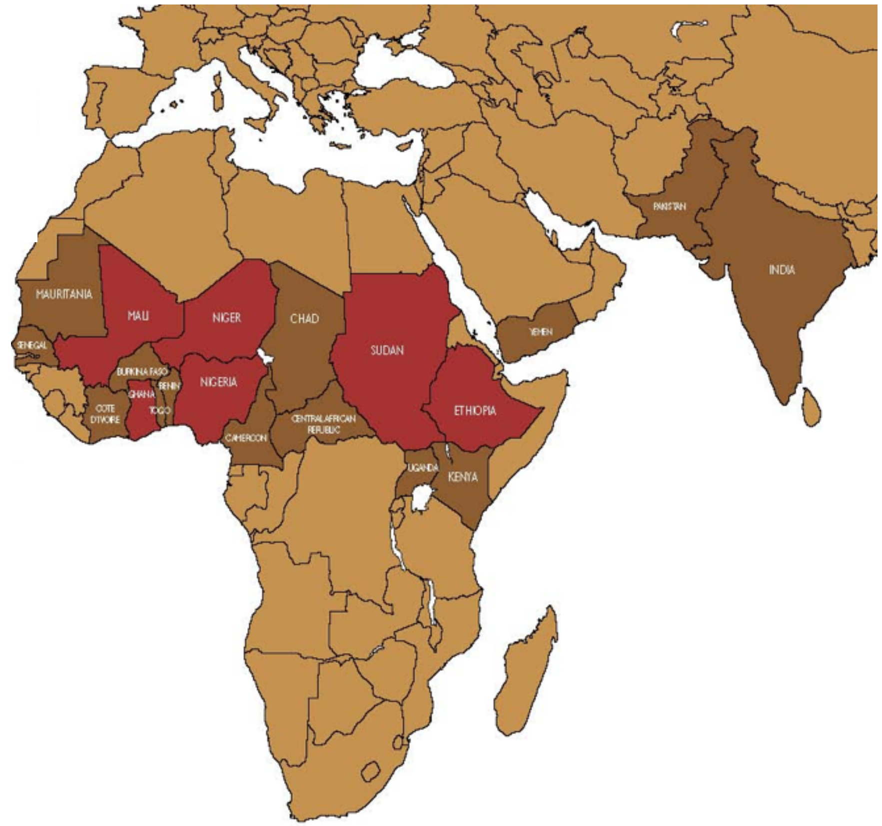

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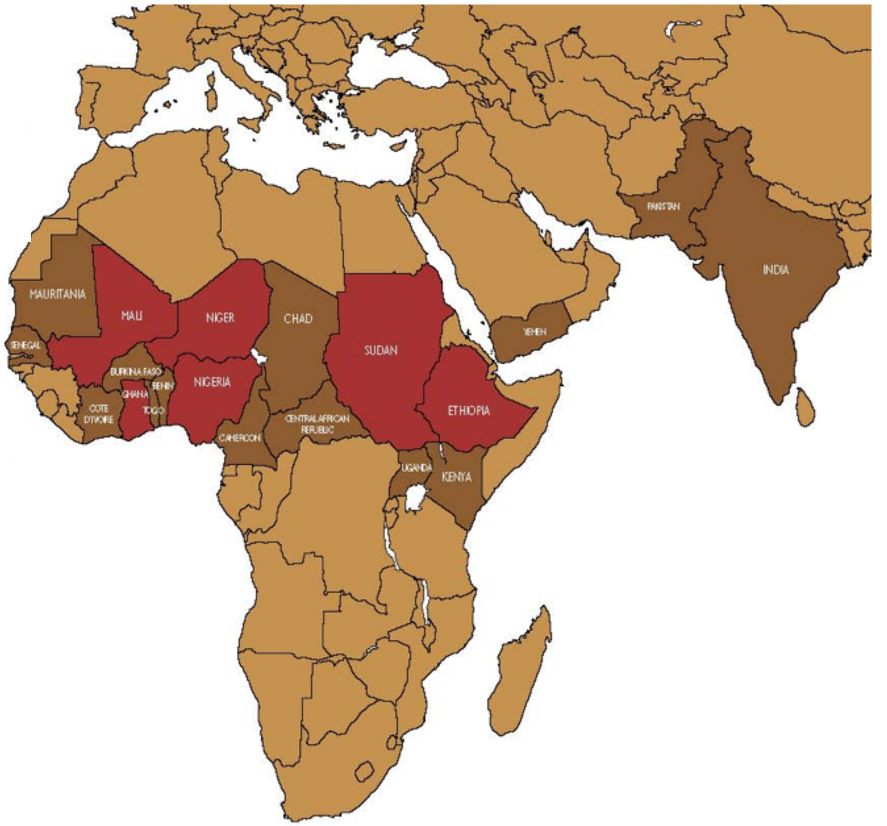

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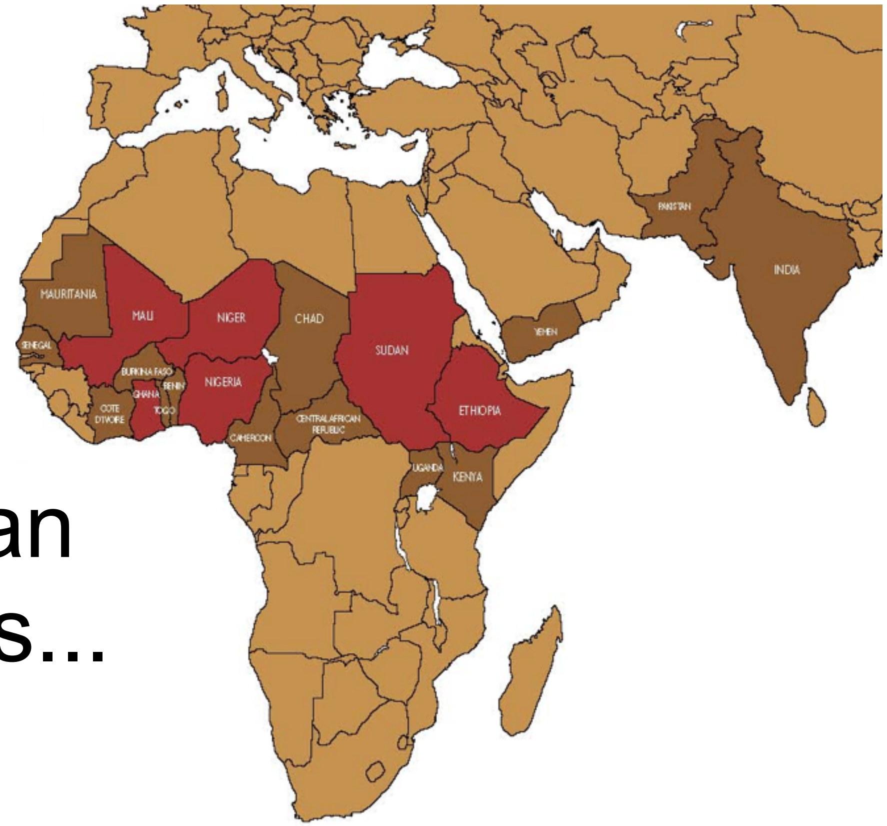

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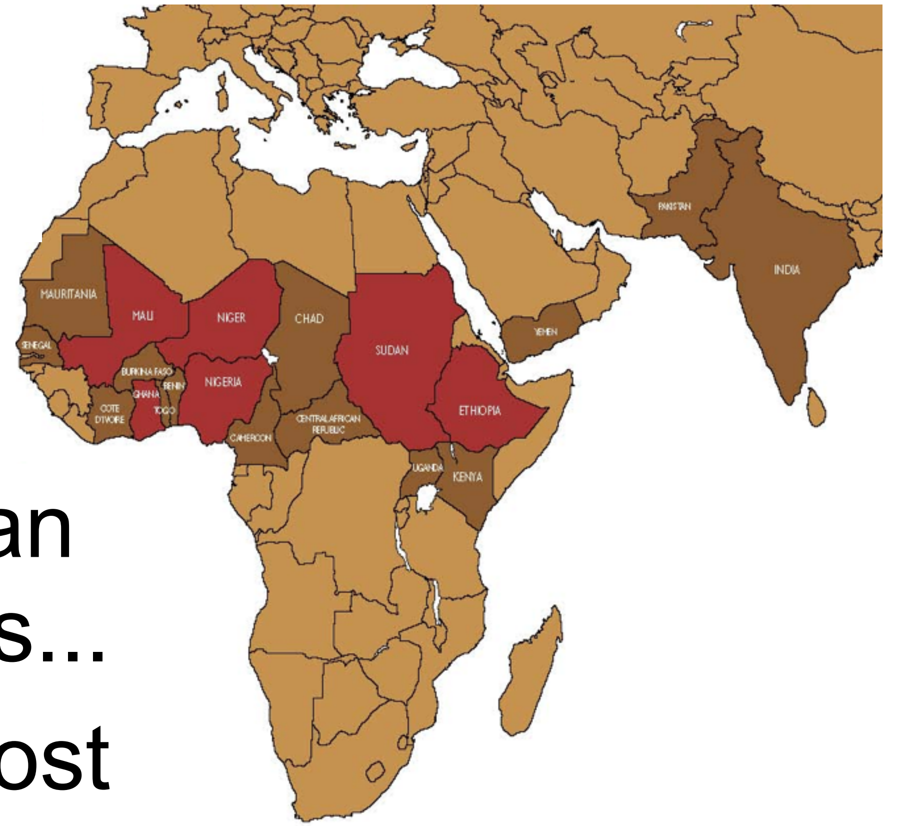
- ...however, today it is almost eradicated.



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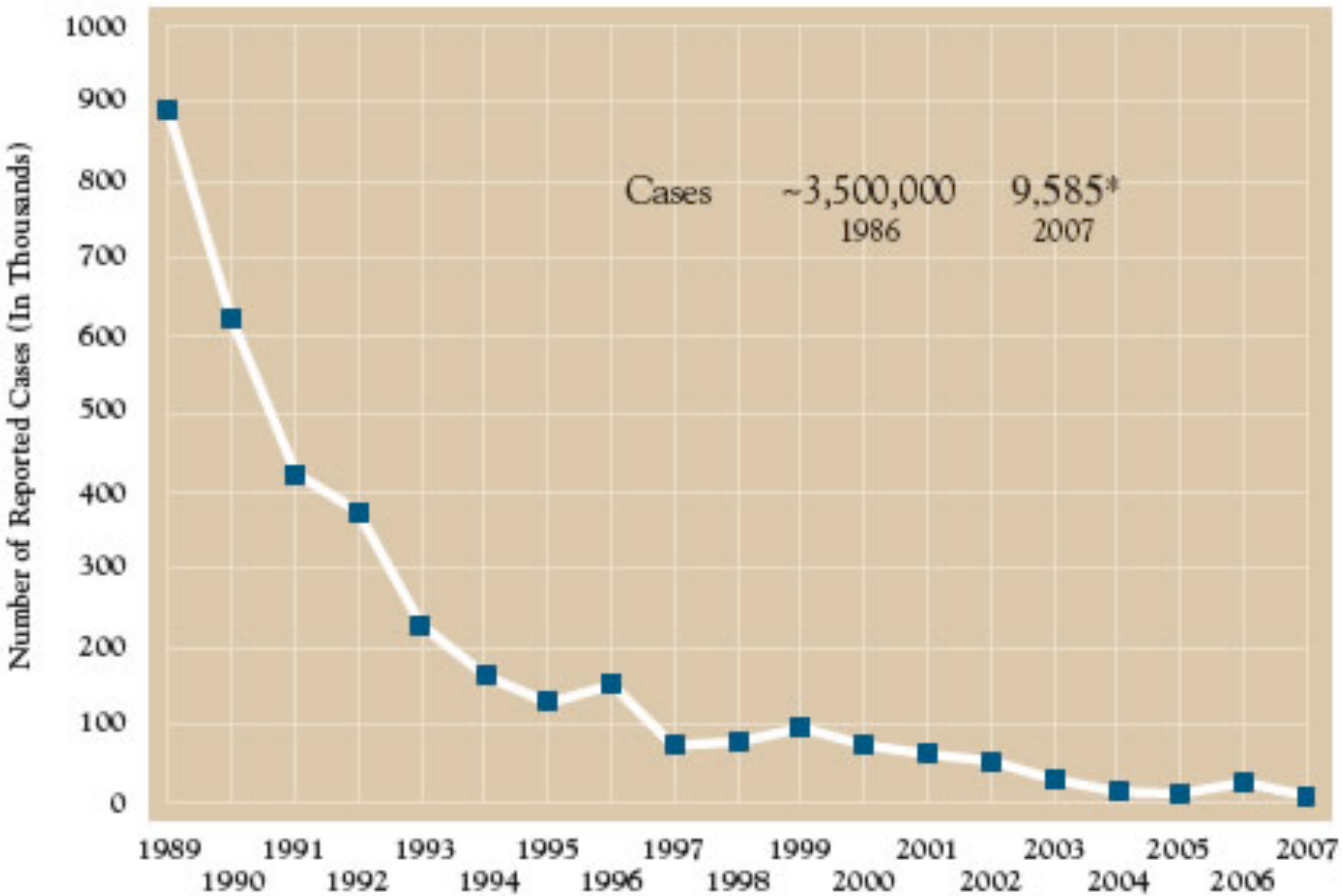
- If eradicated, it will be the first parasitic disease and also the first to be eradicated using behaviour changes alone.

Significant decline



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- Infected individuals can be educated about not submerging wounds in drinking water
- Cloth filters that fit over pots and pans can be distributed to villages
- Nomadic people have received personal-use cloths fitted over pipes, worn around the neck
- Chemical larvacides can be added to stagnant water supplies.

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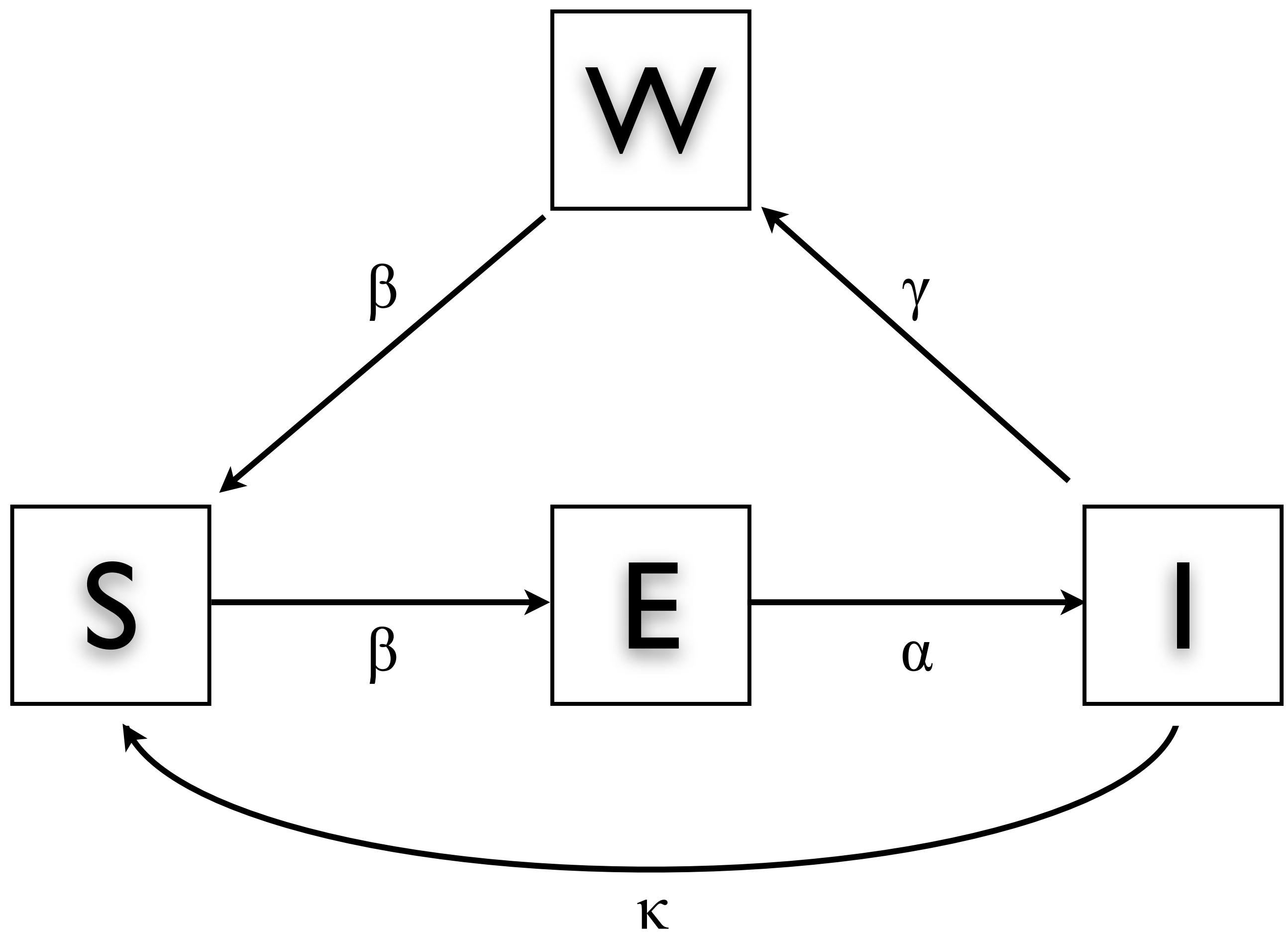
- Thus, we consider chlorination at discrete times.

The model



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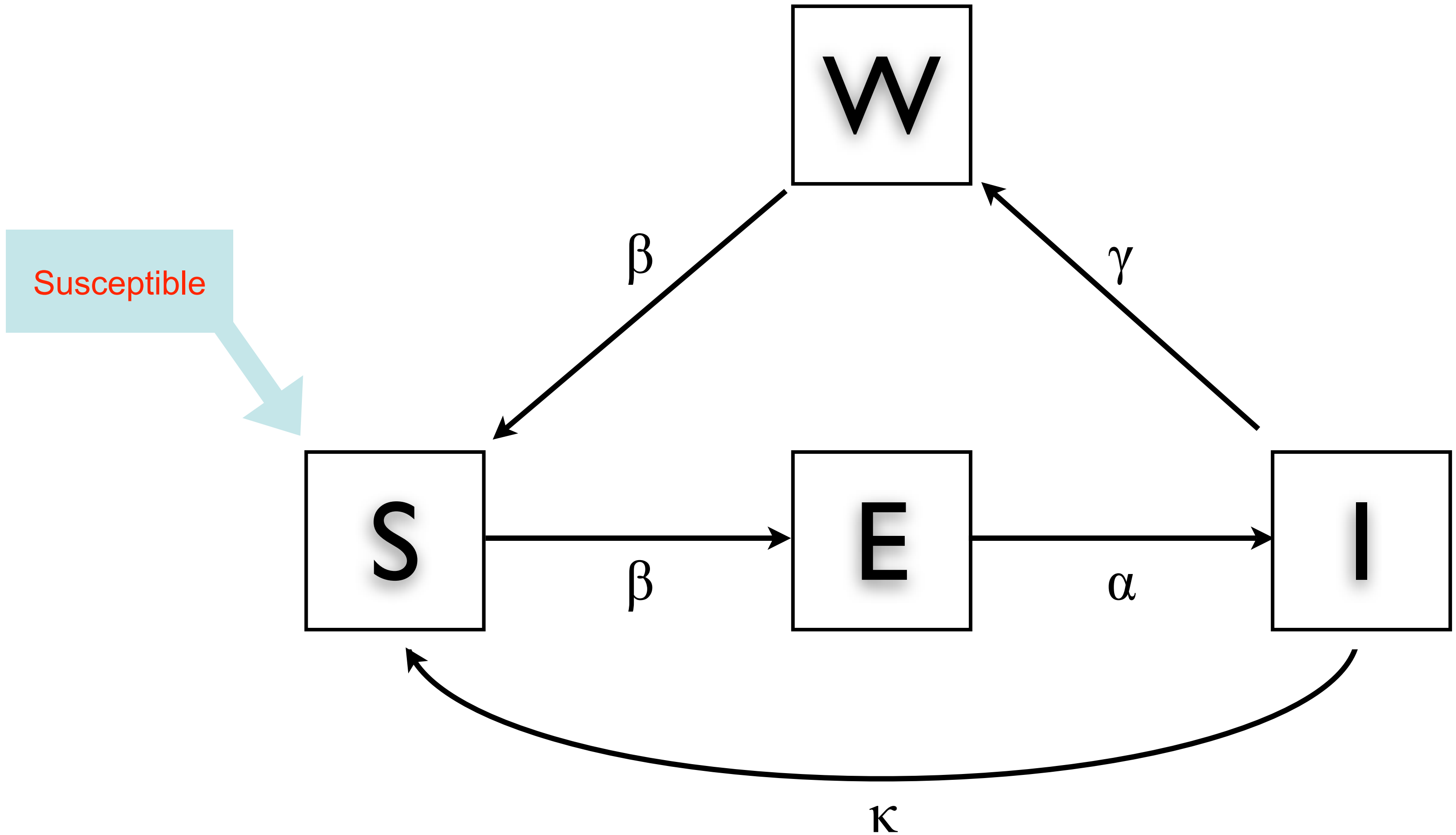

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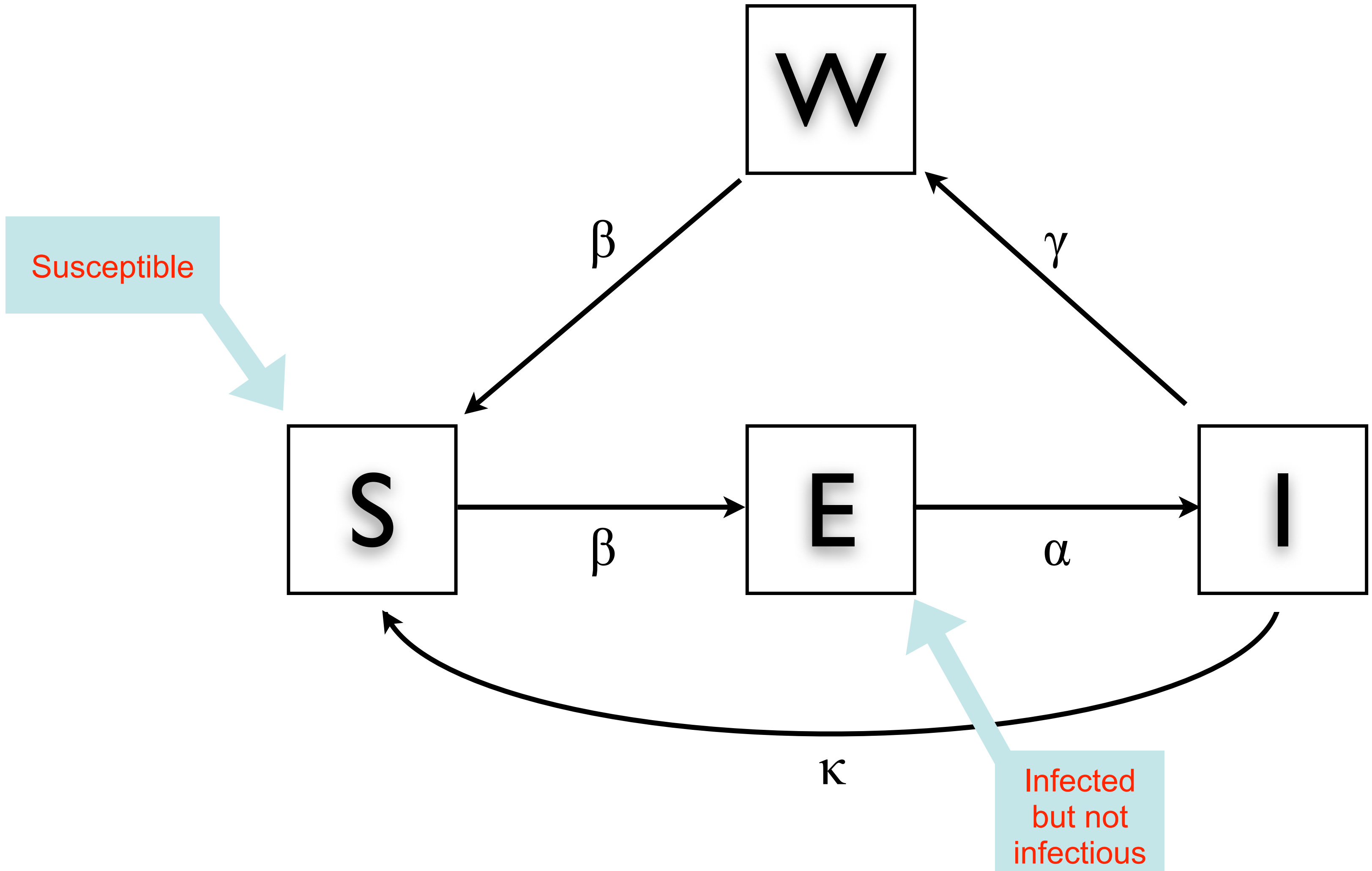

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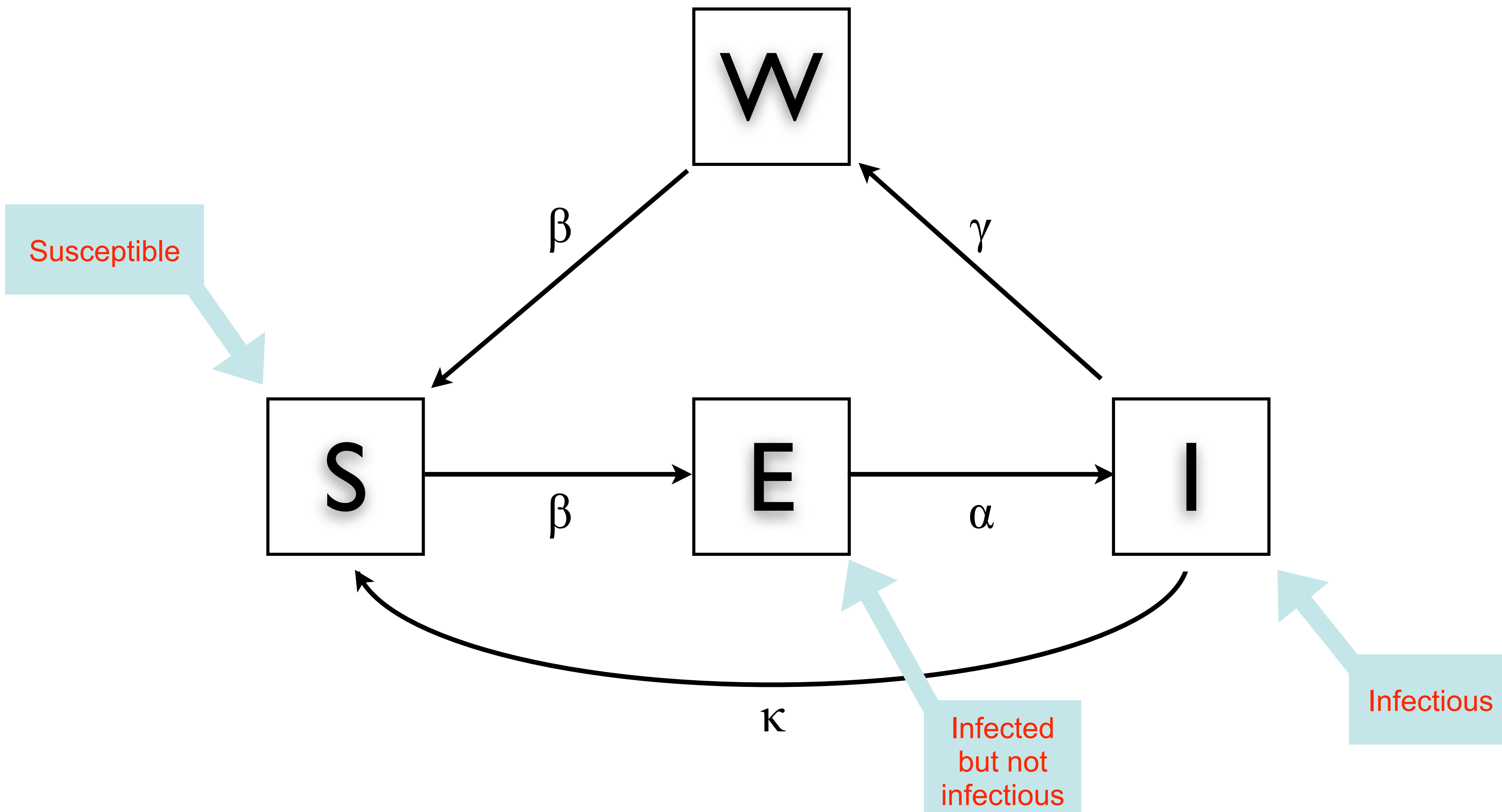

W
β
γ
Susceptible
S
E
I
β
α
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Infected
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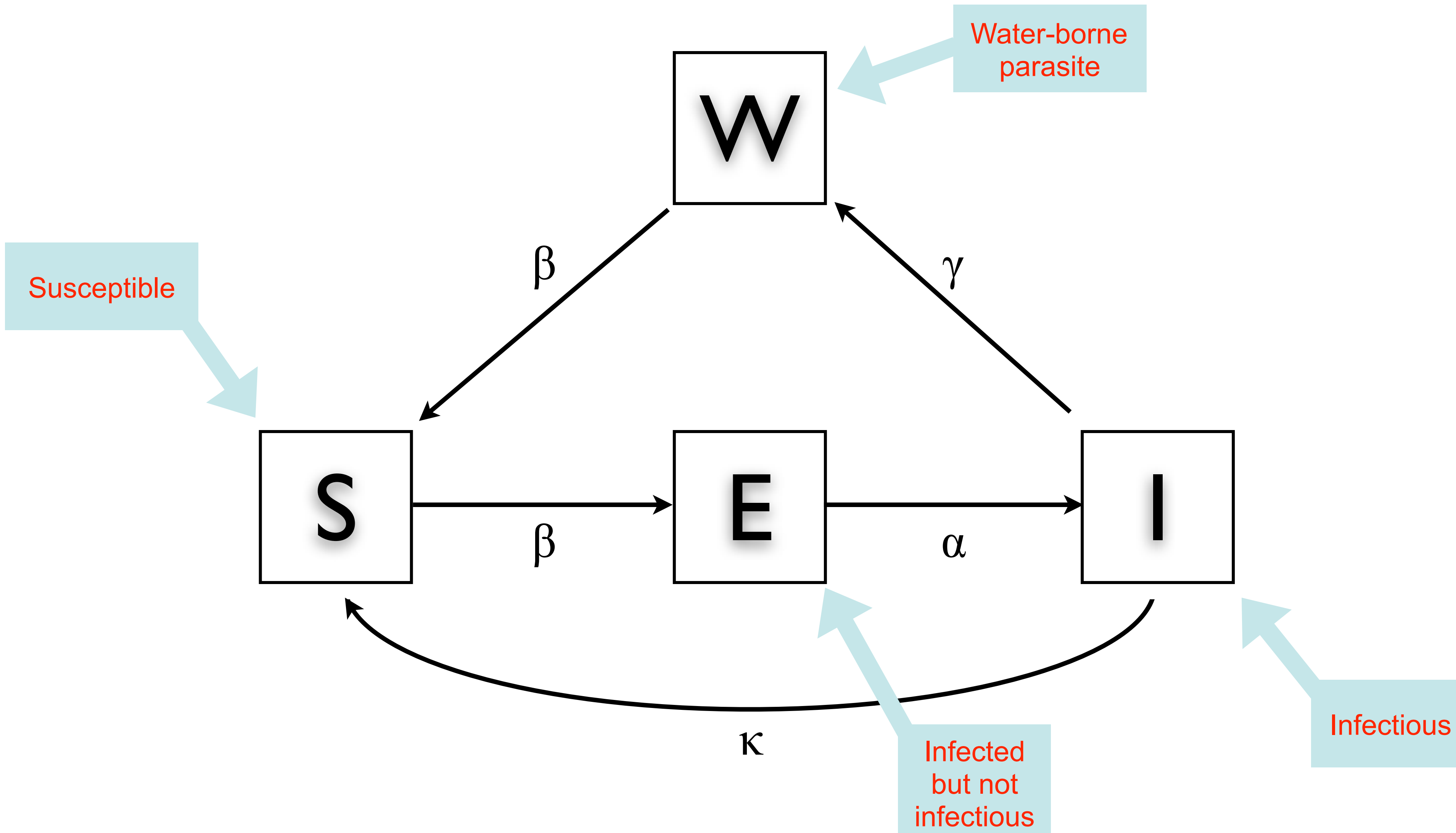
Water-borne parasite
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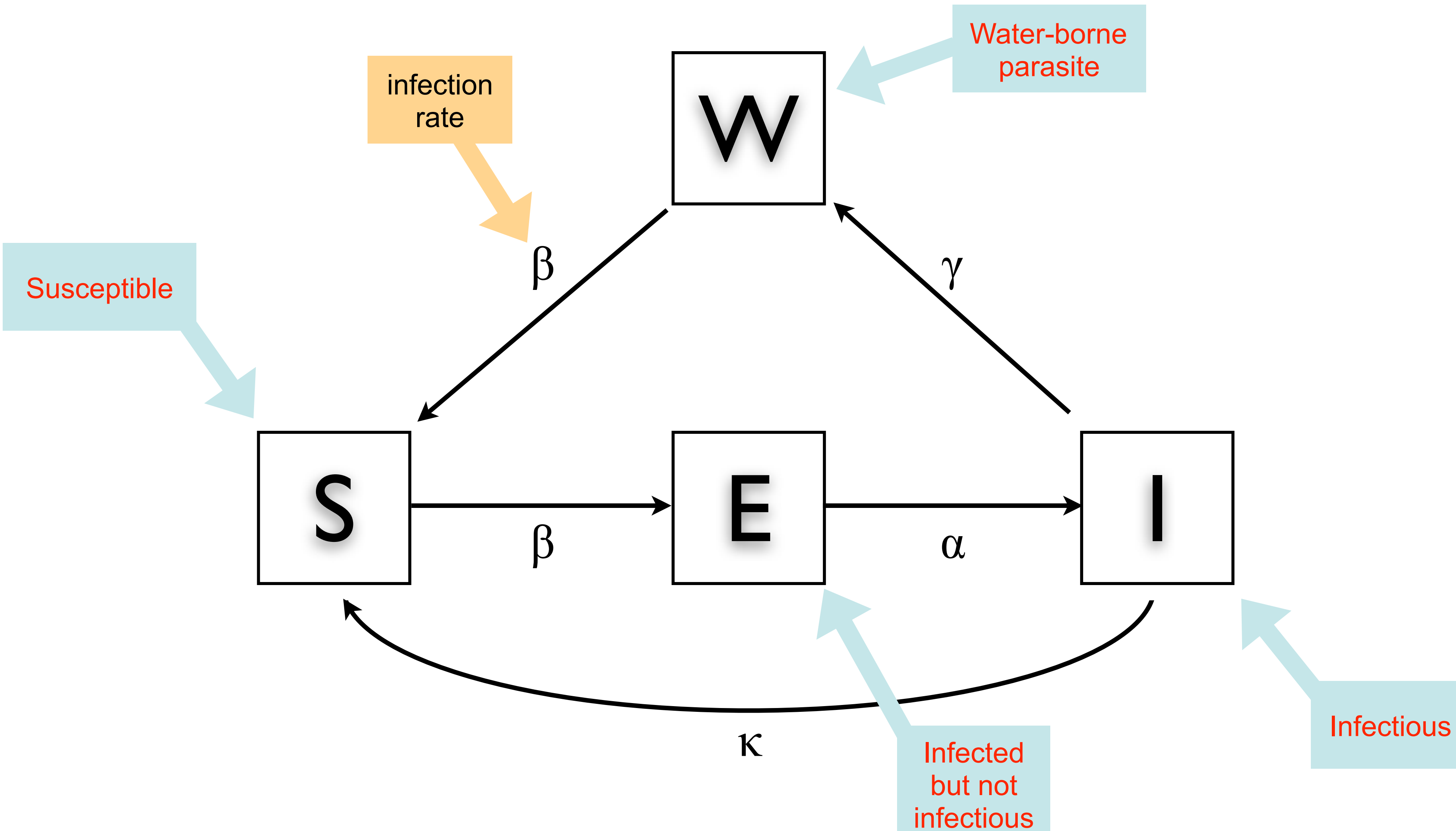

infection rate
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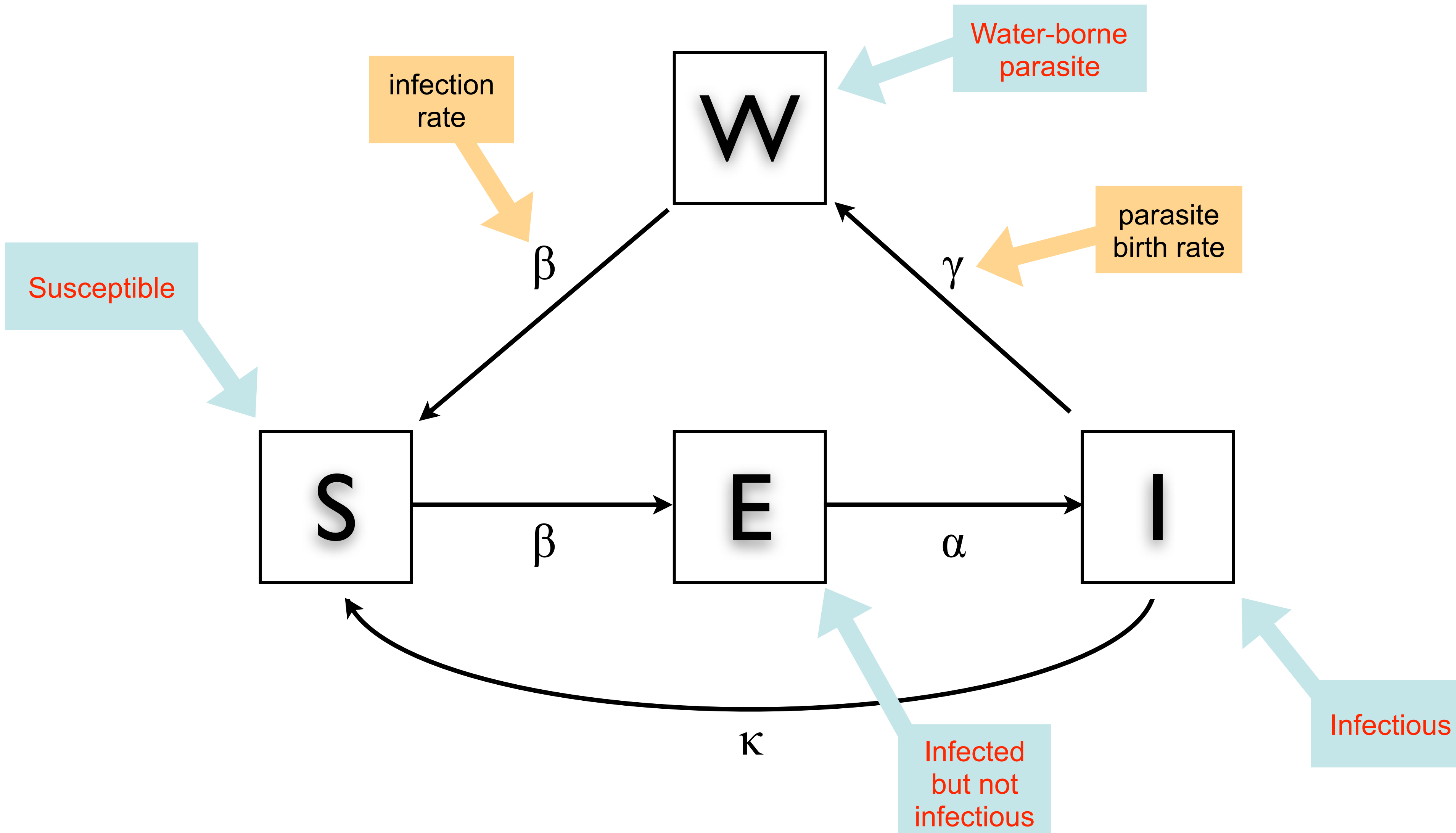
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infection rate
Water-borne parasite
parasite birth rate
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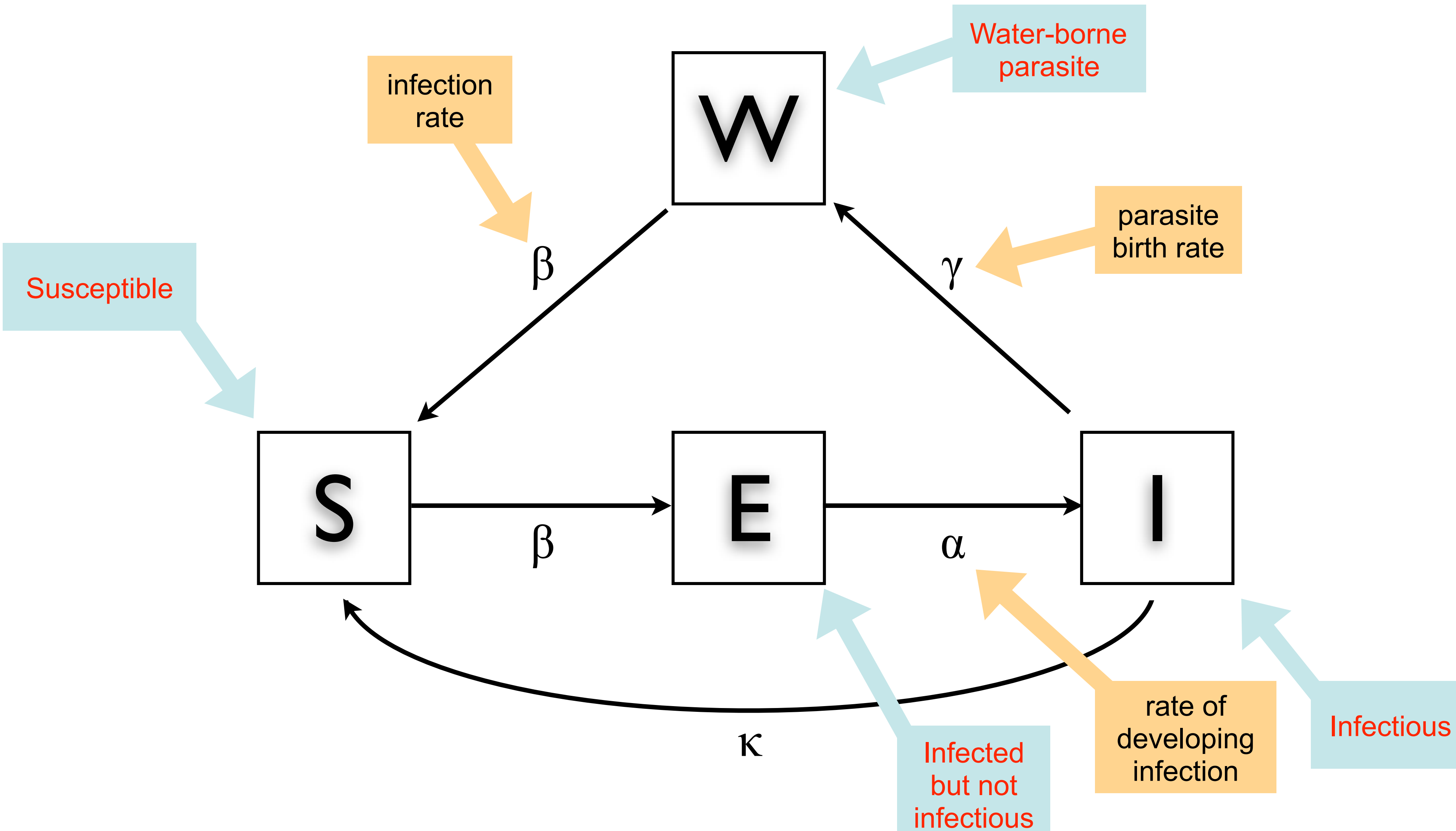

infection rate
W
Water-borne parasite
parasite birth rate
Susceptible
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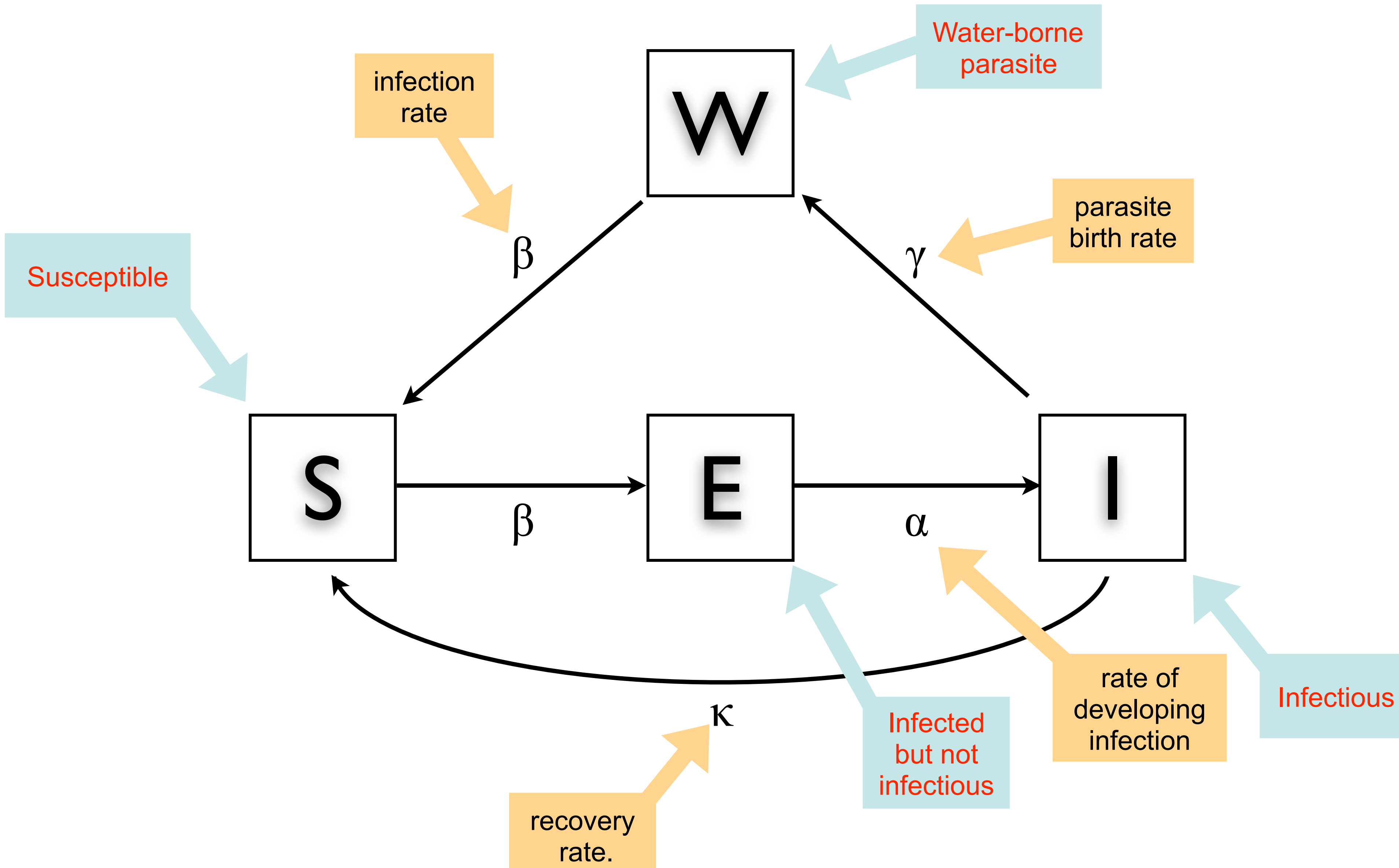
Infected but not infectious
rate of developing infection
Infectious

The model



Water-borne parasite
infection rate
W
parasite birth rate
β
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recovery rate.
Infected but not infectious
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- Impulsive differential equations are a useful formulation for systems that undergo rapid changes in their state
- The approximation is reasonable when the time between impulses is large compared to the duration of the rapid change.

Putting it together

- The model thus consists of a system of ODEs (humans) together with an ODE and a difference equation (parasite).

Equations

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$$
\begin{aligned}
S' &= \Pi - \beta S W - \mu S + \kappa I & t &\neq t_k \\
E' &= \beta S W - \alpha E - \mu E & t &\neq t_k \\
I' &= \alpha E - \kappa I - \mu I & t &\neq t_k \\
W' &= \gamma I - \mu_W W & t &\neq t_k \\
\Delta W &= -rW & t &= t_k
\end{aligned}
$$

S=susceptibles Π=birth rate β=transmissability
μ=background death rate E=exposed I=infectious
W=parasite-infested water κ=recovery rate
α=incubation period γ=parasite birth rate
μ_W=parasite death rate r=chlorine effectiveness

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- t_k is the chlorination time
- Chlorination may occur at regular intervals or not.

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- The former always exists
- The latter only exists for some parameters.

S=susceptibles Π=birth rate
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The basic reproductive ratio

$$R_0 = \frac{\Pi \alpha \gamma \beta}{\mu(\alpha + \mu)(\kappa + \mu)\mu_W}$$

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- Thus, R_0 is our eradication threshold.

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Effect of interventions

- Education discourages infected individuals from putting infected limbs in the drinking water

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- Filtration decreases β and hence R_0
- (Continuous) chlorination increases μ_W and hence decreases R_0.

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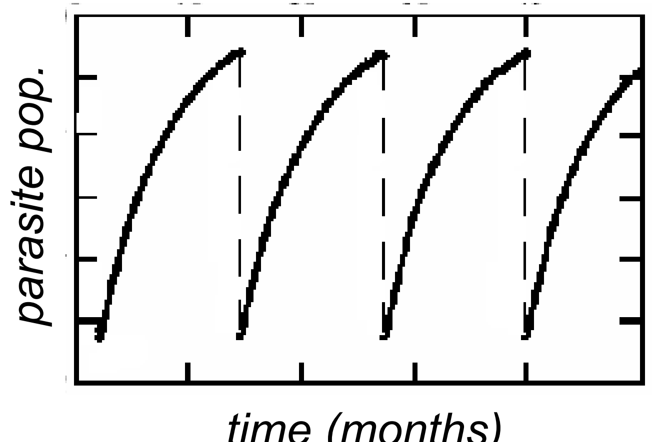

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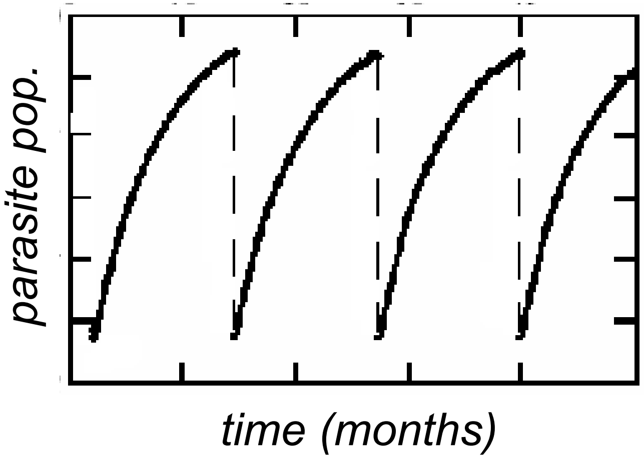
$$W' = \frac{\alpha\Pi\gamma}{\mu(\kappa+\mu)} - \mu_W W \qquad\qquad t \neq t_k$$



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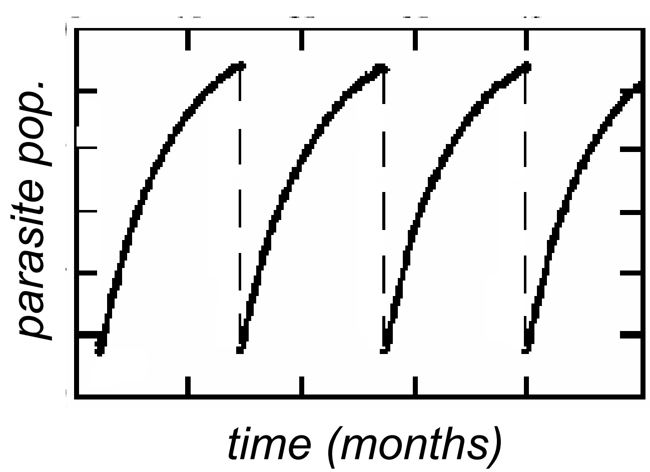

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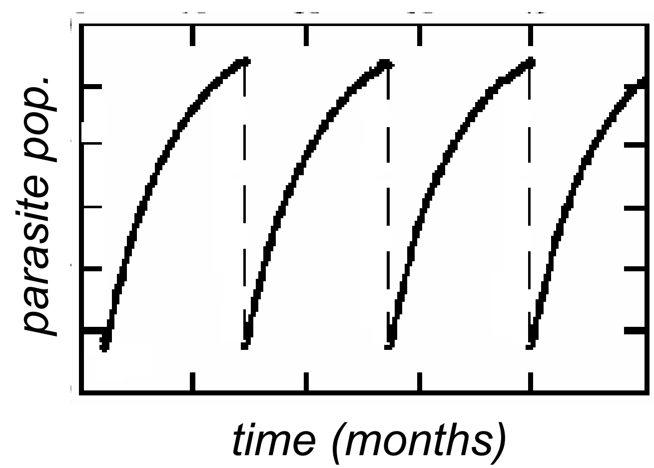
$$W(t_{k+1}^-) = W(t_k^+) e^{-\mu_W (t_{k+1} - t_k)} + \frac{\alpha \Pi \gamma}{\mu \mu_W (\kappa + \mu)} \left[1 - e^{-\mu_W (t_{k+1} - t_k)} \right].$$



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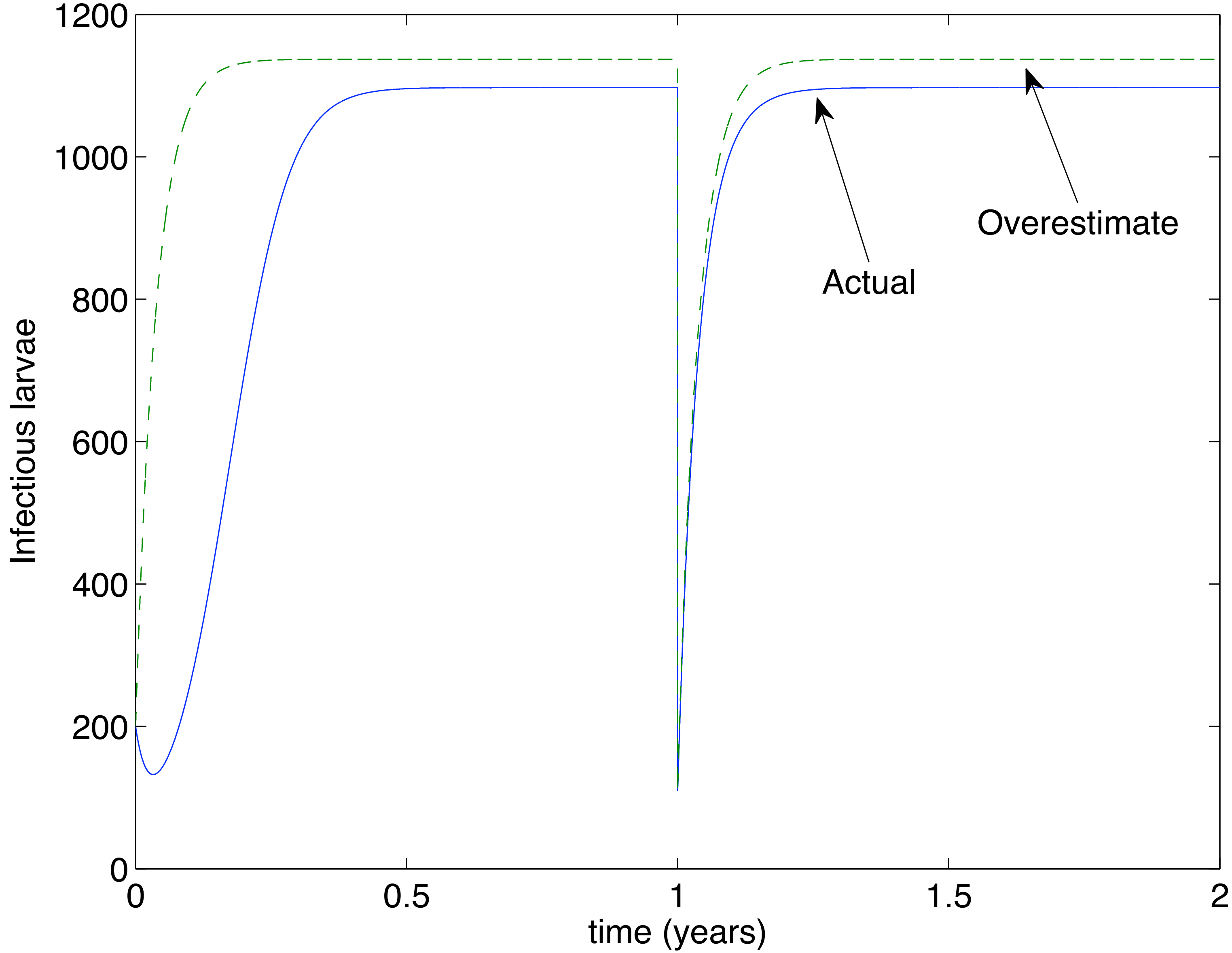
The degree of overestimation



An explicit solution

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$$+\cdots+(1-r)e^{-\mu_W(t_n-t_{n-1})}+1$$
$$-(1-r)^{n-2}e^{-\mu_W(t_n-t_1)} - (1-r)^{n-3}e^{-\mu_W(t_n-t_2)}$$
$$-\cdots-e^{-\mu_W(t_n-t_{n-1})}\Big].$$

Π=birth rate μ=background death rate
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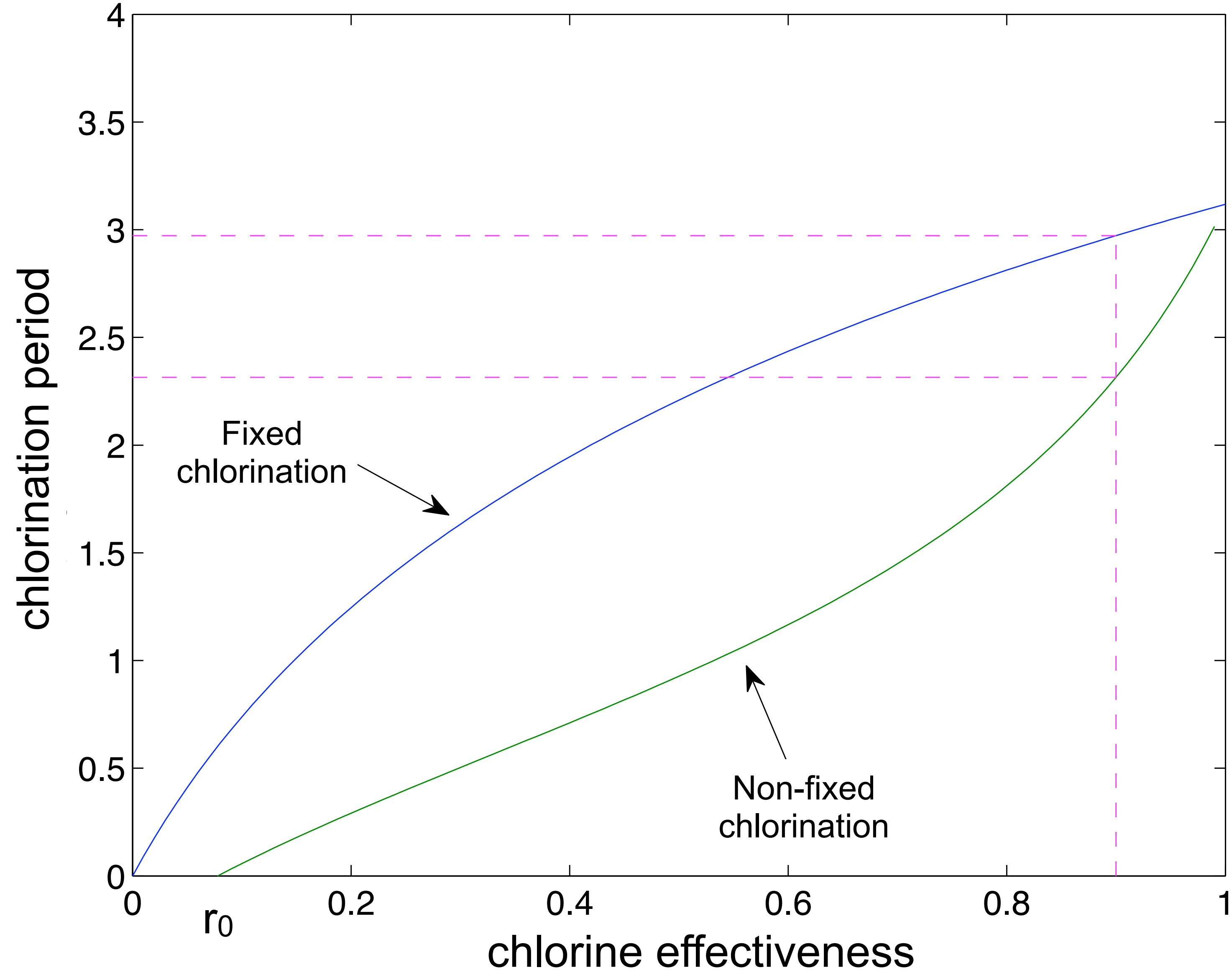
r=chlorination effectiveness

Fixed chlorination is always better



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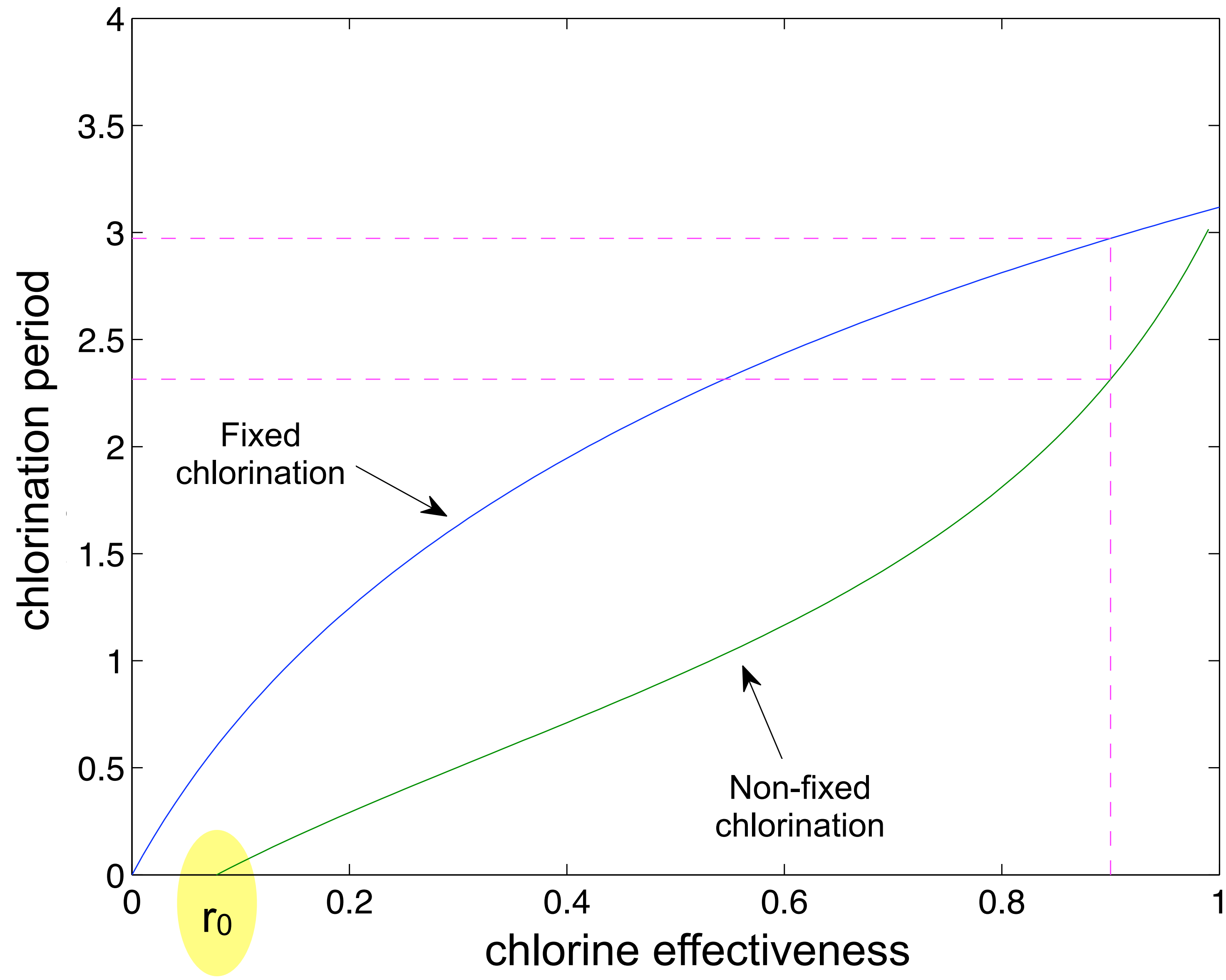

Latin Hypercube Sampling

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  - runs 1,000 simulations.

Example

Example

	✓	

Example

✗	✓	✗
	✗	
	✗	

Example

✗	✓	✗
	✗	✓
	✗	

Example

✗	✓	✗
✗	✗	✓
	✗	✗

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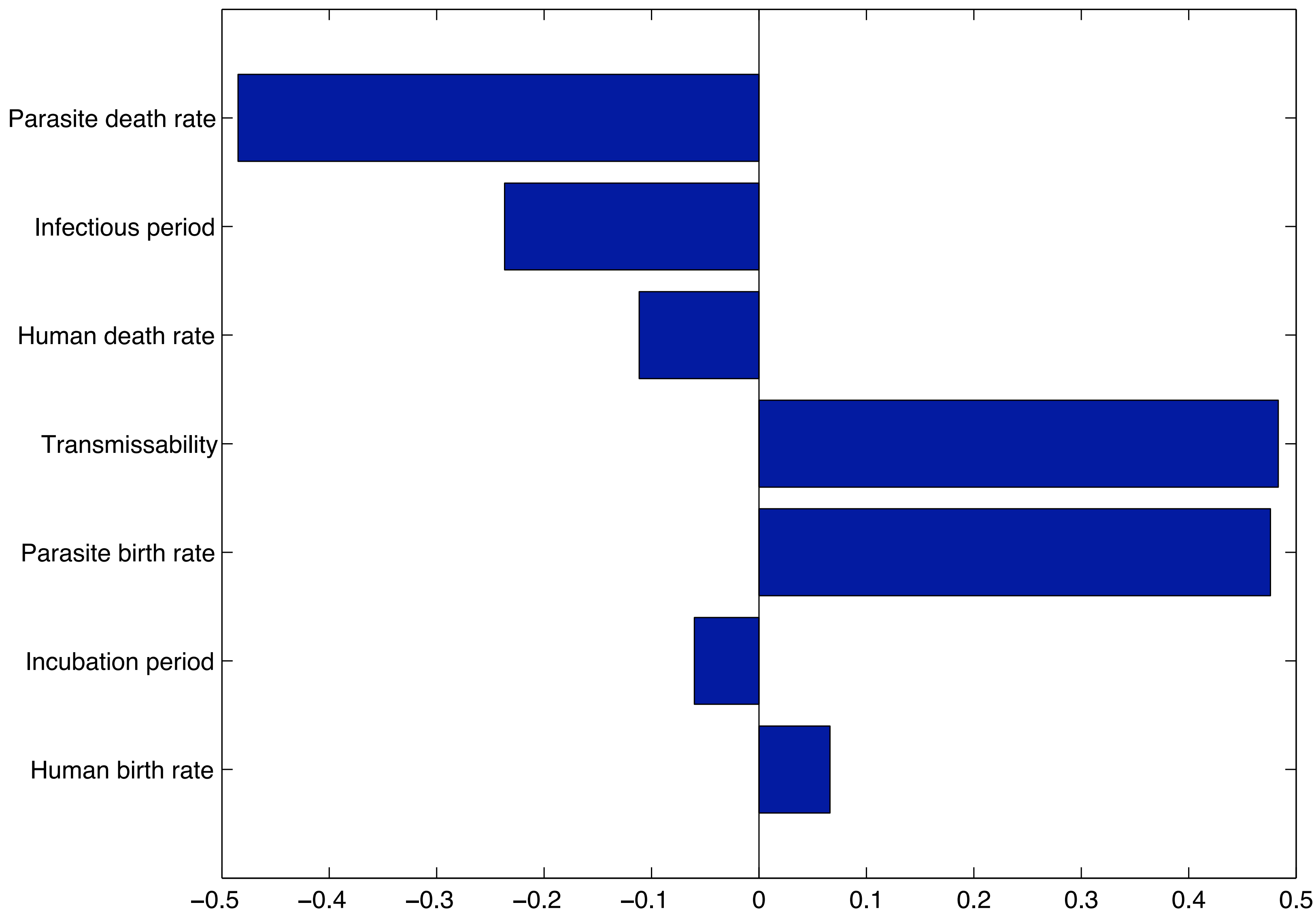
PRCCs


Parasite death rate
Infectious period
Human death rate
Transmissability
Parasite birth rate
Incubation period
Human birth rate
−0.5
−0.4
−0.3
−0.2
−0.1
0
0.1
0.2
0.3
0.4
0.5

Most important parameters

- The three parameters with the most impact on R_0 are

R_0=basic reproductive ratio

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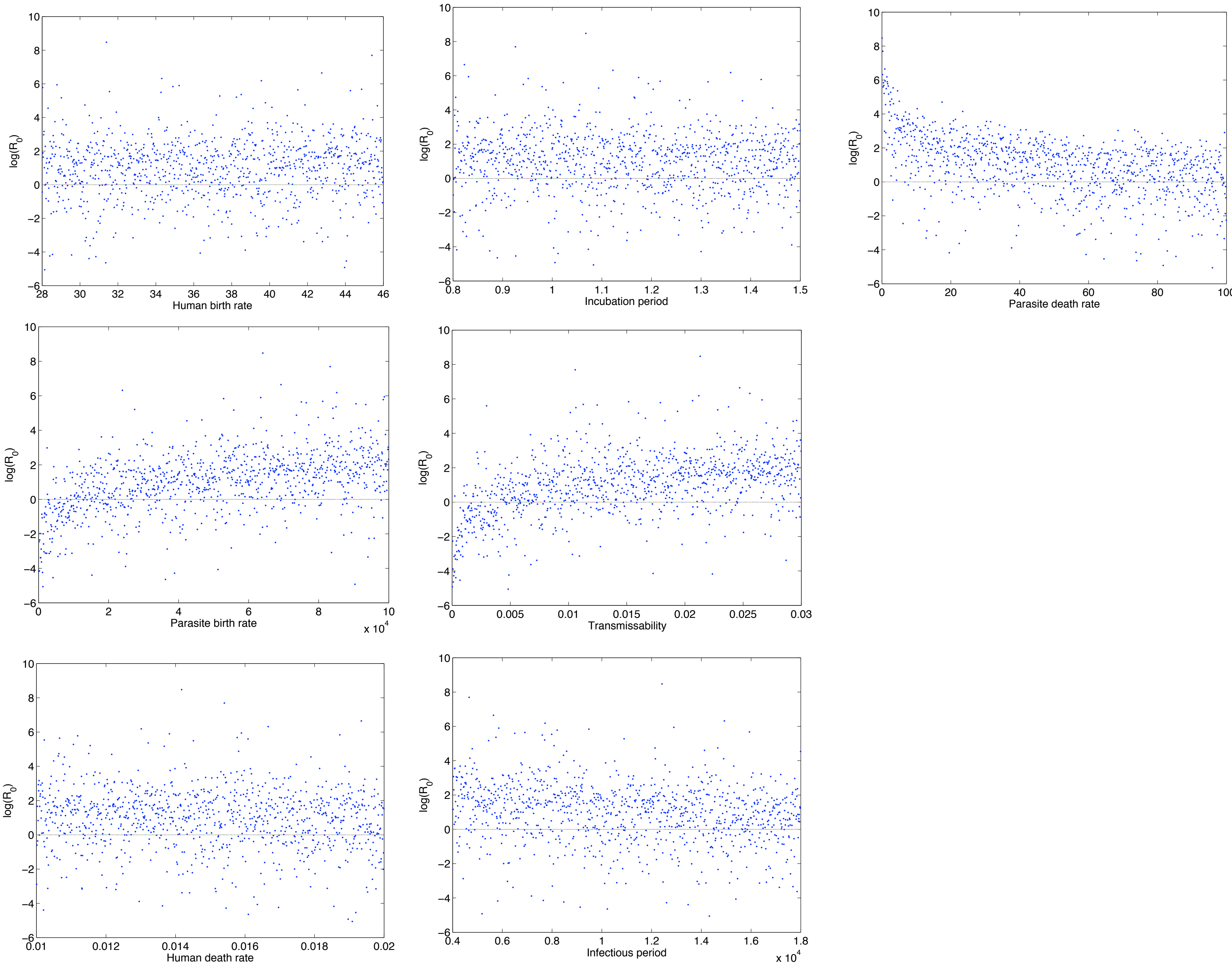
log(R0)
Human birth rate
Incubation period
Parasite death rate
Parasite birth rate
x 10^4
Transmissability
Human death rate
Infectious period
x 10^4

Variation of control parameters

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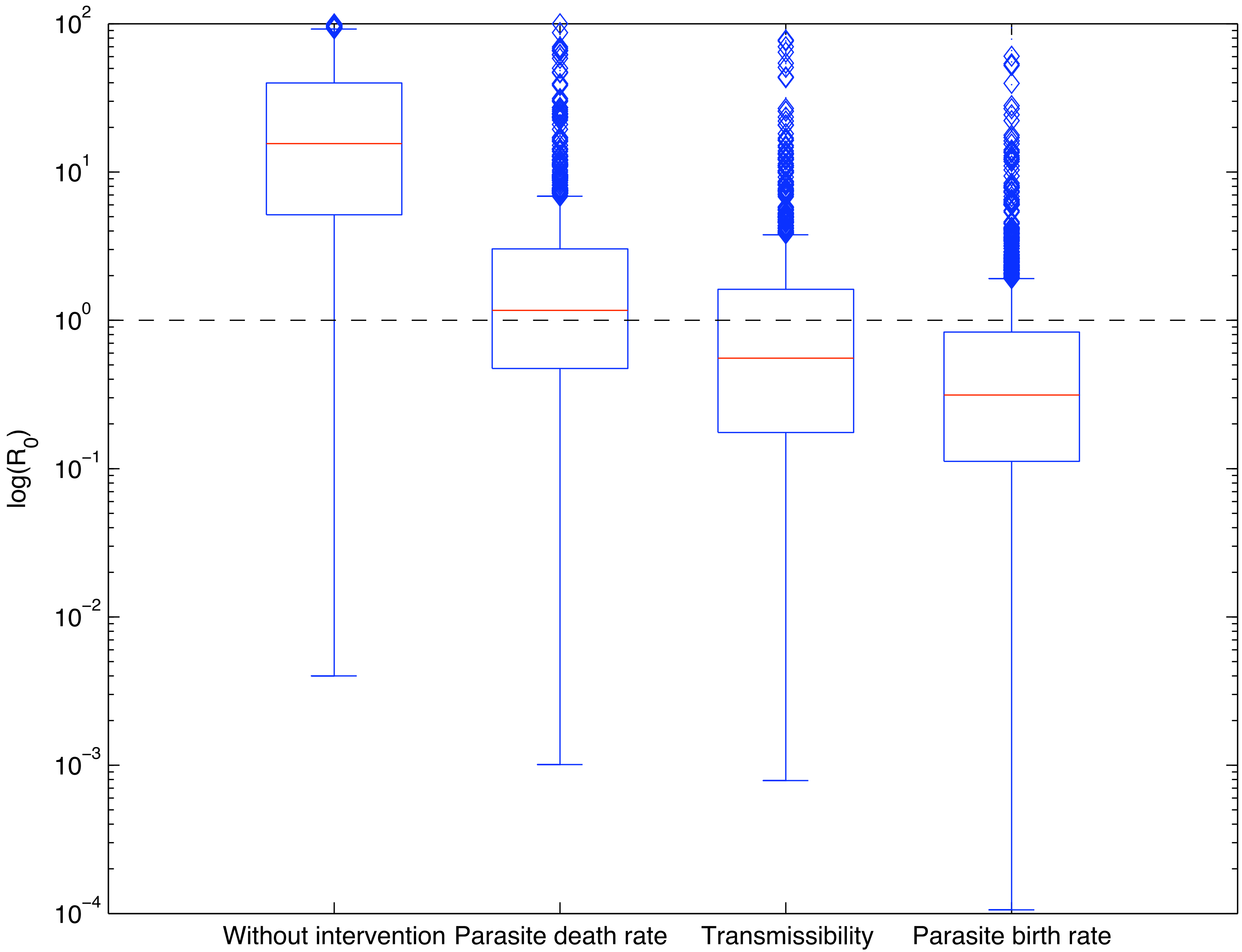
- However, increasing μ_W (eg via continuous chlorination) is unlikely to lead to eradication
- Conversely, sufficiently decreasing γ (via education) is likely to bring R_0 below 1.

γ=parasite birth rate μ_W=parasite death rate R_0=basic reproductive ratio

Altering parameters by a factor of 100



Eradication threshold

- For R_0=1, we can plot the threshold surface for our three control parameters

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  (representing education, filtration and chlorination)

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- We fixed all other parameters at median values.

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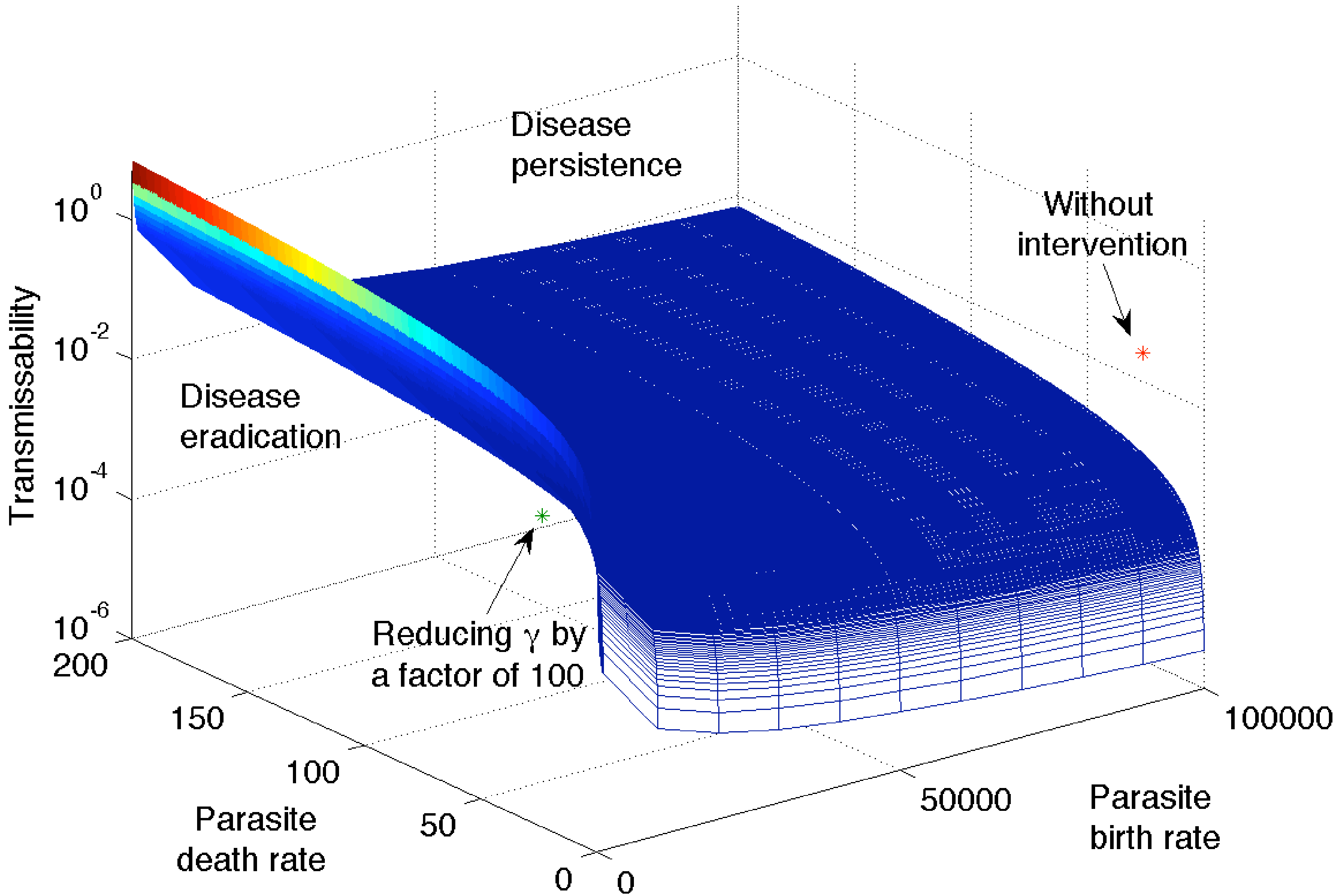
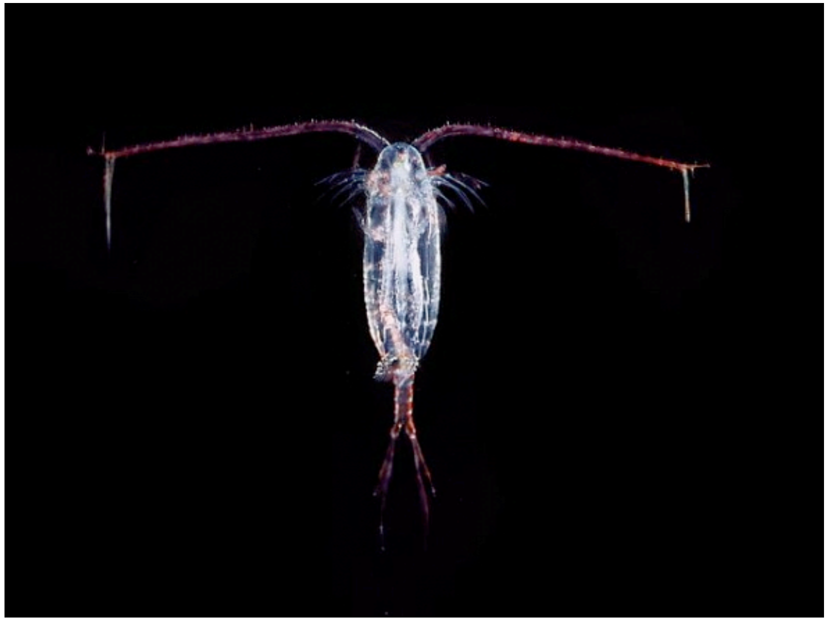


Effect of control parameters

- The outcome is significantly dependent on changes in γ



β=transmissability
γ=parasite birth rate
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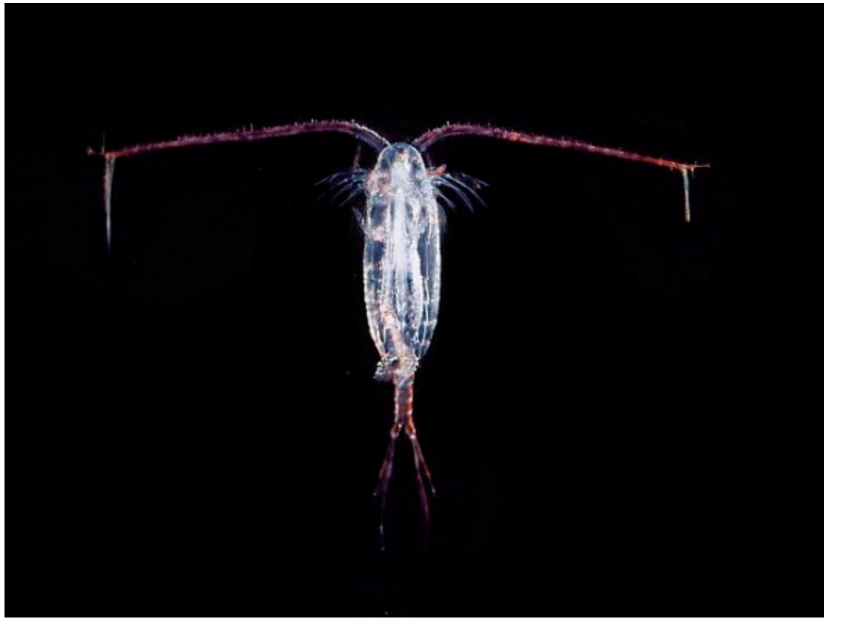

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Effect of control parameters

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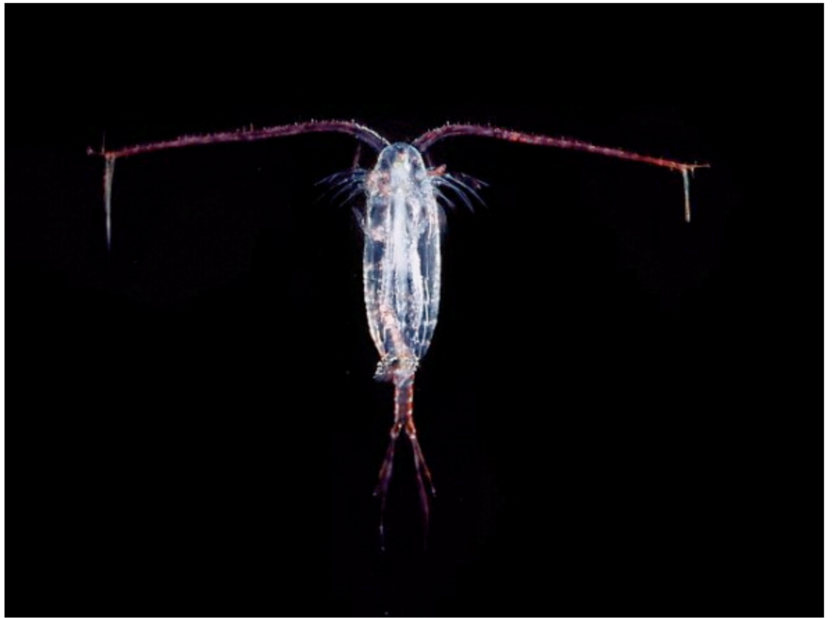
- β would have to be reduced to extremely low levels.



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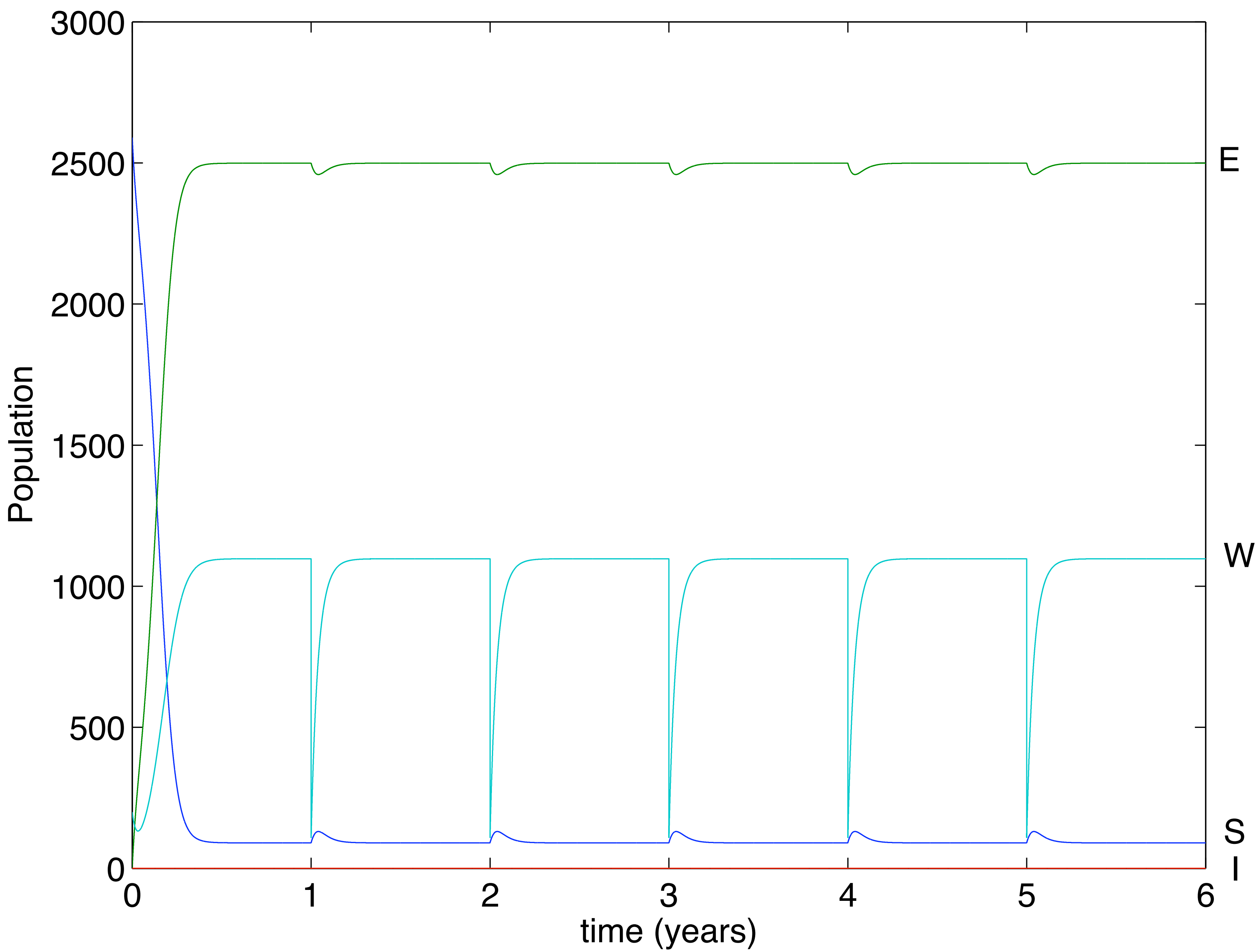
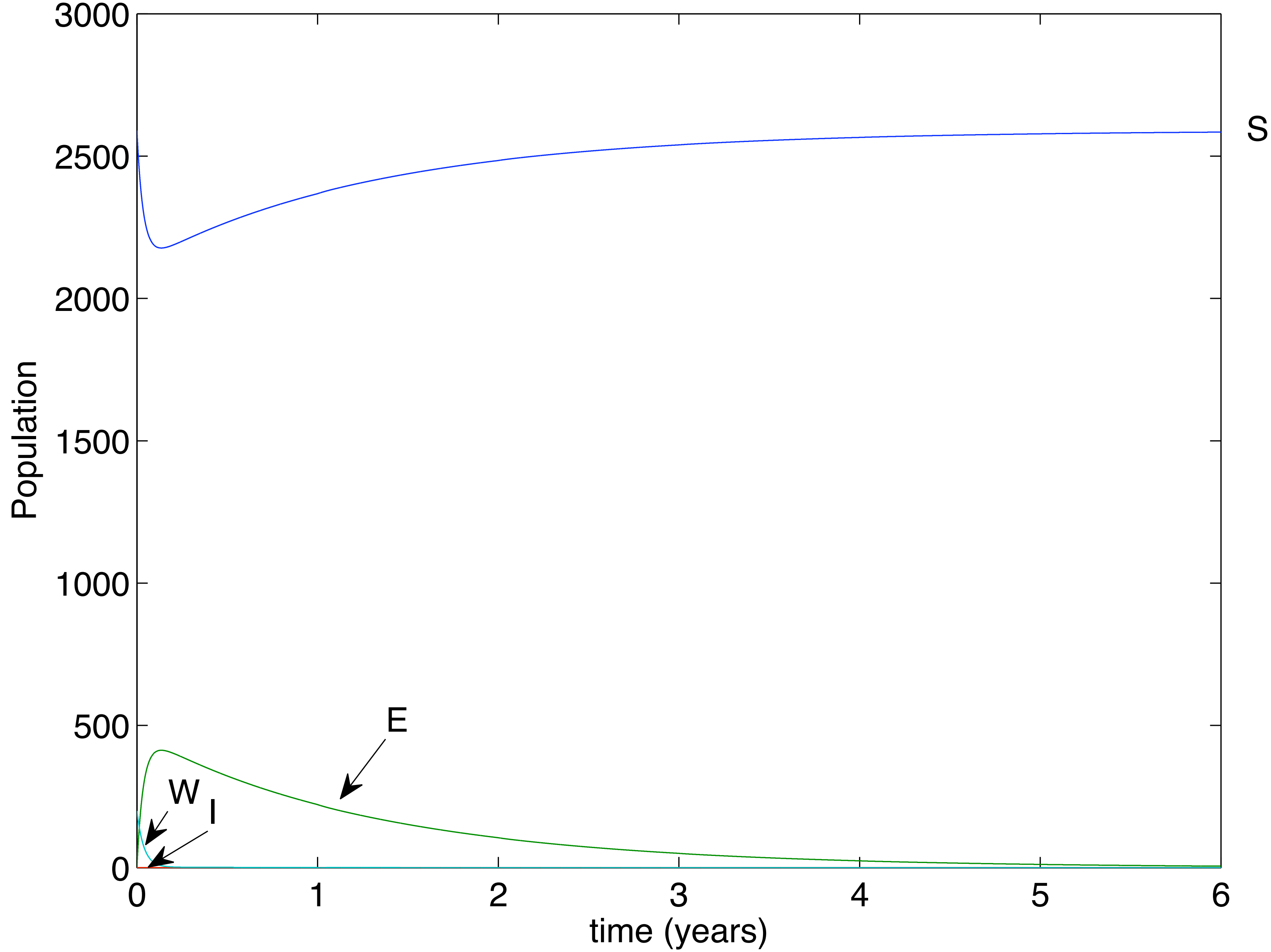
Annual chlorination



Reducing the parasite birthrate by 99%



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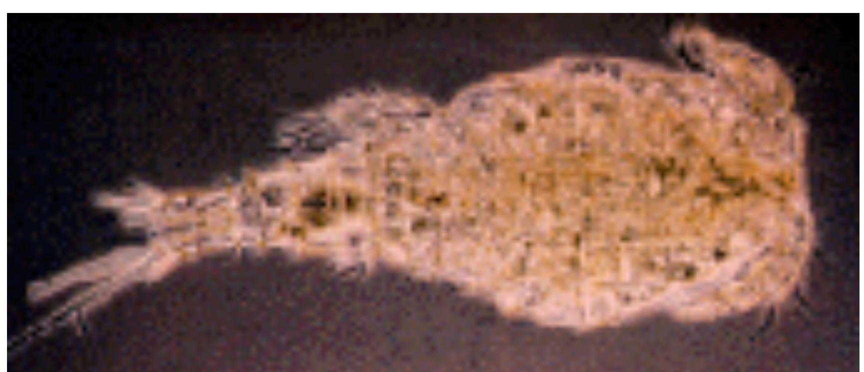
- The entire population becomes uninfected.



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- Guinea worm disease satisfies all three.

Comparison with smallpox

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- However, ongoing efforts mean India was recently declared yaws-free.

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- Education — persuading people not to put infected limbs in the drinking water — is the best way to eradicate Guinea worm disease
- Of course, a combination of education, chlorination and filtration is most desirable
- Efforts should be focussed on reaching remote communities.

Conclusion

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- Without a vaccine or drugs, behaviour changes alone will likely lead to eradication of the first parasitic disease
- This may reshape our understanding of what it takes to eradicate a disease
- By mustering both scientific and cultural resources, we can successfully defeat one of the oldest diseases in human history.

Key references

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